

Semi-parametric and Parametric Inference of Extreme Value Models for Rainfall Data

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Abstract Extreme rainfall events and the clustering of extreme values provide fundamental information which can be used for the risk assessment of extreme floods. Event probability can be estimated using the extreme value index (γ) which describes the behavior of the upper tail and measures the degree of extreme value clustering. In this paper, various semi-parametric and parametric extreme value index estimators are implemented in order to characterize the tail behavior of long-term daily rainfall time series. The results obtained from different estimators are then used to extrapolate the distribution function of extreme values. Extrapolation can be employed to estimate the occurrence probability of rainfall events above a given threshold. The results indicated that different estimators may result in considerable differences in extreme value index estimates. The uncertainty of the extreme value estimators is also investigated using the bootstrap technique. The analyses showed that the parametric methods are superior to the semi-parametric approaches. In particular, the likelihood and Two-Step estimators are preferred as they are found to be more robust and consistent for practical application.

Keywords Extreme rainfall · Extreme value index · Semi-parametric and parametric estimators · Generalized Pareto Distribution

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1 Introduction

The statistical analysis of extreme rainfall events is a prerequisite for water resources risk assessment and decision making. Recent studies indicate changes in intensity and frequency of extremes across the globe (IPCC 2001, 2007; Kharin and Zwiers 2000; Colombo et al. 1999; Goldstein et al. 2003). Changes in extreme climate events (e.g. extreme precipitation, floods, drought, etc.) are particularly important due to their significant impacts on human livelihood and socio-economic developments. In the past twenty five years, five billion people were affected by natural disasters resulting in approximately \$1 trillion in economic losses around the world (Stromberg 2007). Floods, in particular, are the most life threatening hydrologic phenomena that result in extensive property damage and loss of lives each year. Analysis of extreme precipitation events may improve assessment of flood forecast which is influential in water resources management. By analyzing the trend in the annual precipitation values, Karl et al. (1996) reported an increase in the frequency of high intensity precipitation events across the United States over the period of 1910 to 1996. Kharin and Zwiers (2000) highlighted possible future changes in extremes of daily temperature and their effects on extreme precipitation events. Furthermore, several climate studies confirmed the importance of extreme precipitation events on assessment of the impact of potential climate change on the water balance and runoff processes (Abdulla et al. 2009; Iglesias et al. 2007; Xu et al. 2004; Göncü and Albek 2009; Li et al. 2009).

Extreme Value Theory (EVT) is frequently used in water resources engineering and management studies (Katz et al. 2002; Smith 2001; Coles 2001 and references therein) to obtain probability distributions to fit maxima or minima of data in random samples, as well as to model the distribution excess above a certain threshold. For the first time, Fisher and Tippett (1928) introduced the asymptotic theory of extreme value distributions. Gnedenko (1943) provided mathematical proof to the fact that under certain conditions, three families of distributions (Gumbel, Fréchet and Weibull) can arise as limiting distributions of extreme values in random samples. The combination of Gumbel, Fréchet and Weibull families is known as the Generalized Extreme Value (GEV) distribution (Gumbel 1958; Smith 2001; Goldstein et al. 2003). Based on this theory, Gumbel (1942) addressed frequency distribution of extreme values in flood analysis. Gumbel (1958) further developed the GEV theory and applied it for various practical applications. In hydrology literature, the GEV approach is often referred to as the *annual maxima* or *block maxima* when it is used to fit distributions to maxima/minima of a given dataset (Goldstein et al. 2003).

An alternative approach to the *annual maxima* is the so-called *threshold* method which is based on exceedances above thresholds. The *threshold* approach is the analog of the generalized extreme value distribution for the annual maxima, but it leads to a distribution called the Generalized Pareto Distribution (GPD) which is proven to be more flexible than the annual maxima (Smith 2001; Goldstein et al. 2003; Smith and Shively 1995). One advantage of the *threshold* methods over the *annual maxima* is that they can deal with asymmetries in the tails of distributions (McNeil and Frey 2000).

The fundamentals of extreme value analysis based on threshold values were established by Balkema and De Haan (1974) and Pickands (1975) whereby the

mean number of exceedances above a high threshold in a cluster was given as the key parameter. Extreme event probability can be estimated using the extreme value index (γ) which describes the behavior of the upper tail and measures the degree of clustering of extreme values. The rainfall extreme value index corresponds to the mean number of exceedances above a high threshold in a cluster, and can be estimated using a limited sample from an unknown distribution. For practical application of the GPD approach, the parameters are to be estimated first. Hill (1975) and Pickands (1975) introduced the Hill and the Pickands estimators respectively. Dekkers et al. (1989), Danielsson and Vries (1997) and Ferreira et al. (2003) proposed and applied different moment based estimators for the extreme value index. Landwehr et al. (1979) presented the Probability Weighted Moments (PWM) approach, that is linear combinations of L-moments (Hosking 1990), for parameter estimation of extreme value distributions. The L-moment theory offers a parameter estimation technique which is particularly used when the sample size is limited (Kharin and Zwiers 2000). Laurini and Tawn (2003) introduced a method known as the two-threshold approach whereby the extreme value index is represented by the number of independent clusters observed in the sample data. This method is limited in that it requires the preliminary identification of clusters, and the choice of two declustering parameters. Zhou (2008) proposed a two-step scale invariant maximum likelihood estimator of the extreme value index and Ferro and Segers (2003) adopted a moment-based estimator for high quantiles. However, the latter is not flexible in analyzing changes of the extreme value index over time. Lu and Peng (2003) developed an empirical likelihood method for the extreme value index of heavy tailed distributions. In addition to the quantitative parameter estimation methods, mentioned above, there are several graphical techniques such as the quantile-quantile (Q-Q) plot and probability plot correlation coefficient (PPCC) that can be used for data exploration and visual goodness-of-fit tests (Goldstein et al. 2003).

Recently, application of extreme value theory has drawn the attention of hydrologists for prediction of extreme events. Different longterm rain gauge and surface runoff data have been investigated with respect to their extreme value distributions (see Pagliara et al. 1998; Bernardara et al. 2008; Withers and Nadarajah 2000; Haylock and Nicholls 2000; Koutsoyiannis and Baloutsos 2000; Aronica et al. 2002; Nguyen et al. 2002; Segal et al. 2002; Crisci et al. 2008; Shane and Lynn 1964; Todorovic and Zelenhasic 1970; Madsen et al. 1997a, b; Alila 1999; Fowler and Kilsby 2003; Koutsoyiannis 2004; Semmler and Jacob 2004; Martins and Stedinger 2000; Morrison and Smith 2002; Glaves and Waylen 1997). For other applications of the extreme value theory in meteorology, oceanography and climatology the interested reader is referred to Khaliq et al. (2005), van den Brink et al. (2004), Voss et al. (2002), van den Brink et al. (2003), Sánchez-Arcilla et al. (2008) and Galiatsatou et al. (2008). Ouarda et al. (1994) presented commonly used extreme value distributions in hydrology based on the asymptotic behavior of their probability density functions. Egozcue and Ramis (2001) investigated heavy precipitation events across the eastern Spain and reported heavy tailed distributions for the extremes. Buishand (1991) conducted analyses of precipitation extremes over regional scales. In general, the objective of extreme rainfall analysis is to obtain reasonable estimates of extreme rainfall quantiles which exceed a given threshold. As discussed previously, this can

be achieved by fitting the Generalized Pareto (GP) distribution to the excesses over a certain threshold. In this paper, different extreme value index estimators are implemented in order to characterize the tail behavior of long-term daily rainfall time series. The results obtained from different estimators are then used to extrapolate the distribution function of extreme values. Such extrapolation can be employed to estimate occurrence probability of rainfall above a certain threshold. In this study, the uncertainty of the extreme value index estimators is investigated using the bootstrap technique (Efron 1979, 1981, 1987; Davison and Hinkley 1997; Carpenter and Bithell 2000; Shao and Tu 1995; Efron and Tibshirani 1997). This technique is a data-driven method that employs Monte Carlo simulations to draw random sub-samples from the data under consideration in order to generate an empirical estimate for the sampling distribution of a statistic (Qi 2008). The bootstrap has been frequently used in practical applications for determining the accuracy (or uncertainty) of statistical analyses (Dunn 2001; Kharin and Zwiers 2000; Dixon 2002; Khaliq et al. 2005; Ferro and Segers 2003; Zwiers 1990; Zhang et al. 2005; Wilks 1997; Zwiers and Ross 1991).

The paper is divided into seven sections. After the introduction, the required theoretical background to follow the subsequent sections is briefly reviewed. The third section discusses various extreme value index estimators. In the fourth section, properties and restrictions of the estimators are reviewed. The fifth section describes rainfall data and the study areas. The sixth section is devoted to the results and discussion whereas the last section summarizes the findings and conclusions.

2 Theoretical Background

Generally, the steps to analyze the tail behavior of extremes based upon the Generalized Pareto Distribution can be summarized after Pérez-Fructuoso et al. (2007): (1) Selecting a threshold over which the GPD is fitted. (2) Parameter estimation using an extreme value index estimator which can reasonably model the tail behavior. (3) Evaluating the goodness of fit. (4) Performing the inference of extreme value models. This paper is devoted mainly to parameter estimation using various extreme value index estimators (step 2) and their associated uncertainties.

Theoretically, extreme value distributions are derived as limiting distributions of large or small values in samples from random variables. Performing a linear transformation in order to reduce the actual size of large (or small) values is essential to obtain a non-degenerate limiting distribution (see Kotz and Nadarajah 2000). Keeping this in mind, let F be the distribution function of x_1, x_2, \dots, x_n independent and identically distributed (i.i.d.) random variables. For a non-degenerate distribution function G , $a_n > 0$ and $b_n \in \Re$ the following argument is valid:

$$G(x) = \lim_{n \rightarrow \infty} F^n(a_n x + b_n) \quad (1)$$

where a_n and b_n may depend on the sample size but not on x . Equation 1 indicates that for all continuity points x of G , $F \in D(G)$ (F is in the domain of attraction of the distribution G) and thus, $[G(x)]^n = G(a_n x + b_n)$. For a complete review and details, see: Qi (2008), Fisher and Tippett (1928), Galambos (1978) and Zhou (2008).

Different representations of a_n and b_n result in various extreme-value distributions. For example, the Gumbel distribution is obtained by assuming $a_n = 1$ whereas Fréchet and negative Weibull are derived by taking $a_n \neq 1$. For derivations, readers are referred to Qi (2008) and Galambos (1978). The behavior of extreme values are commonly described by the above mentioned distributions whose cumulative distribution functions are displayed below:

$$G(x) = e^{-e^{-x}} \tag{2}$$

$$\Phi(x) = \begin{cases} 0 & x \leq 0 \\ e^{-x^{-\alpha}} & x > 0 \text{ and } \alpha > 0 \end{cases} \tag{3}$$

$$W(x) = \begin{cases} e^{-(-x^{-\alpha})} & x < 0 \text{ and } \alpha < 0 \\ 1 & x \geq 0 \end{cases} \tag{4}$$

where: α = distribution parameter
 x = a sequence of independent and identically distributed (i.i.d) random variables

The combination of the above distribution families is known as the generalized extreme value (GEV) distribution that can be used to approximate the maxima of finite sequences of random variables. The standard cumulative distribution function of the GEV can be expressed as:

$$\Psi(x) = \exp \left\{ - \left(1 + \gamma \left(\frac{x - \mu}{\sigma} \right) \right)^{-\frac{1}{\gamma}} \right\} \tag{5}$$

where: μ = location parameter
 σ = scale parameter
 γ = shape parameter

The extreme value index (γ), also known as the shape parameter, governs the tail behavior of the GEV distribution. The function $\Psi(x)$ is defined for $1 + \gamma \left(\frac{x - \mu}{\sigma} \right) > 0$; elsewhere, the function $\Psi(x)$ is either 0 or 1 (Smith 2001). This implies, for $\gamma > 0$ or $\gamma < 0$, the density function has zero probability above (below) the upper (lower) bound defined as $-1/\gamma$. In the limit as γ approaches 0, the GEV distribution is unbounded. In Eq. 5, $\gamma = 0$, $\gamma < 0$ and $\gamma > 0$ represent the Gumbel, Weibull (with $\alpha = 1/\gamma$) and Fréchet (with $\alpha = 1/\gamma$) families, respectively. In the GEV distribution, if the $\gamma > 0$, the distribution is heavy tailed. The standard form of the GEV ($G_\gamma(x) = e^{-(1+\gamma x)^{-1/\gamma}}$) can be obtained by substituting $\mu = 0$ and $\sigma = 1$ into Eq. 5.

Note that the mean and standard deviation of the GEV distribution exist if $\gamma < 1$ and $\gamma < 1/2$, respectively (Smith 2001):

$$M_X = E(X) = \mu - \frac{\sigma}{\gamma} + \frac{\sigma}{\gamma} (\Gamma(1 - \gamma)) \tag{6}$$

$$S_X = \sqrt{E(X - M_X)^2} = \frac{\sigma}{\gamma} \sqrt{(\Gamma(1 - 2\gamma) - \Gamma^2(1 - \gamma))} \tag{7}$$

where $M_X(\gamma < 1)$ and $S_X(\gamma < 1/2)$ are the mean and standard deviation. Equations 6 and 7 can be extended to the fact that the n th moment of the GEV distribution exists if $\gamma < 1/n$. In subsequent sections, based upon the above formulation and conditions, various extreme value index estimators are reviewed and applied for rainfall data.

3 Estimation of the Extreme Value Index

There are generally two types of extreme value index estimators: (1) parametric such as the Maximum Likelihood, Probability Weighted Moment, Two-Step and Three-Step estimators; (2) semi-parametric such as the Pickands, Hill and Moment estimators. The main assumption behind parametric estimators is that the data follows an exact GEV probability distribution function, defined by a number of parameters. Semi-parametric estimators, however, are based on partial properties of the underlying distribution function. In the following, extreme value index estimators used in this study are described in details.

3.1 Pickands Estimator

Assume a finite sample of $x_1, x_2 \dots x_n$ from a sequence of i.i.d. random variables satisfying Eq. 1 is observed. Let $x_{n,1} \leq x_{n,2} \leq \dots \leq x_{n,n}$ be the ranking order of $x_1, x_2 \dots x_n$. Then, $x_{n,n} - x_{n,n-k}, \dots, x_{n,n-k+1} - x_{n,n-k}$ is the excess above the threshold where k denotes the rank of the upper threshold. Pickands (1975) introduced an estimator for the extreme value index as follows:

$$\gamma_P = (\log 2)^{-1} \log \left(\frac{x_{n,n-k/4} - x_{n,n-k/2}}{x_{n,n-k/2} - x_{n,n-k}} \right) \tag{8}$$

3.2 Hill Estimator

Using the maximum likelihood approach and the excess ratios $(x_{n,n}/x_{n,n-k}, \dots, x_{n,n-k+1}/x_{n,n-k})$ instead of the excess above the threshold (as mentioned in Pickands Estimator), Hill (1975) proposed the following estimator for $\gamma > 0$:

$$\gamma_H = k^{-1} \sum_{i=1}^k \log (x_{n,n-i+1} - x_{n,n-k}) \tag{9}$$

where: k = rank of the upper threshold

The Hill estimator is based on the asymptotic behavior of the largest order statistics (Hill 1975) and is commonly used on practical applications (e.g. Casson and Coles 1999; Brutsaert and Parlange 1998).

3.3 Moment Estimator

Dekkers et al. (1989) presented a more generalized estimator for $\gamma \in \mathfrak{R}$ known as the Moment estimator:

$$\begin{aligned} \gamma_M &= k^{-1} \sum_{i=1}^k \log(x_{n,n-i+1} - x_{n,n-k}) \\ &+ 1 - \frac{1}{2} \left(1 - \frac{\left(k^{-1} \sum_{i=1}^k \log(x_{n,n-i+1} - x_{n,n-k}) \right)^2}{k^{-1} \sum_{i=1}^k \log(x_{n,n-i+1} - x_{n,n-k})^2} \right)^{-1} \end{aligned} \tag{10}$$

3.4 Probability Weighted Moment Estimator

By assigning different weighting factors to the excesses above the threshold, Hosking and Wallis (1987) suggested the probability weighted moment estimator:

$$\gamma_{WM} = \frac{k^{-1} \sum_{i=0}^{k-1} (x_{n,n-i} - x_{n,n-k}) - 4k^{-1} \sum_{i=0}^{k-1} \frac{i}{k} (x_{n,n-i} - x_{n,n-k})}{k^{-1} \sum_{i=0}^{k-1} (x_{n,n-i} - x_{n,n-k}) - 2k^{-1} \sum_{i=0}^{k-1} \frac{i}{k} (x_{n,n-i} - x_{n,n-k})} \tag{11}$$

3.5 Maximum Likelihood Estimator

Smith (1987) employed the maximum likelihood method to estimate $\gamma \in \mathfrak{R}$. For the case, $\gamma \neq 0$ the maximum likelihood estimator can be expressed by solving the following two equations:

$$\begin{aligned} \gamma_{MLE} &= k^{-1} \sum_{i=0}^{k-1} \log \left(1 + \frac{\gamma_{MLE}}{\sigma} (x_{n,n-i} - x_{n,n-k}) \right), \\ \frac{1}{1 + \gamma_{MLE}} &= k^{-1} \sum_{i=0}^{k-1} \frac{1}{1 + \frac{\gamma_{MLE}}{\sigma} (x_{n,n-i} - x_{n,n-k})} \end{aligned} \tag{12}$$

where: $\sigma =$ extreme value scale

Note that in the estimation of extreme quantiles of the GEV distribution, the likelihood estimator is not reliable for small sample sizes (Hosking and Wallis 1987). This issue is further discussed in Section 4.

3.6 Two-Step Estimator

Zhou (2008) suggested the Two-step estimator as an approximation to the likelihood formulation. The main motivation for this approach is based upon the fact that there is no general explicit formulation for the likelihood estimator. In this approach, the

extreme value index is pre-estimated in the first step using, say, Pickands estimator. In the second step, an approximation to the likelihood function is performed such that the difference between the Two-step and likelihood estimators tend to zero. For proof and details, readers are referred to Zhou (2008).

$$\gamma_{2S} = \frac{2\gamma_P + 1}{2} \frac{WM^2}{(WM)^2} - 1 \tag{13}$$

where:

$$WM^j = \sum_{i=1}^k w_i^j (x_{n,n-i+1} - x_{n,n-k})^j, j = 1, 2 \tag{14}$$

and

$$w_i^j = \frac{1}{j\gamma_P + 1} \left(\left(\frac{i}{k}\right)^{j\gamma_P + 1} - \left(\frac{i-1}{k}\right)^{j\gamma_P + 1} \right), j = 1, 2 \tag{15}$$

3.7 Three-Step Estimator

By taking the final result of the Two-Step estimator and assuming it to be the first estimate of the above procedure, Zhou (2008) proposed the Three-Step estimator, requiring one more iteration (the same as the Two-Step approach) to obtain the extreme value index.

4 Estimator Properties

There are a number of extreme value index estimators, commonly used in practical applications, but few are both shift and scale invariant (Aban and Meerschaert 2001). A shift invariant estimator is analogous to a time-invariant function; defined such that if $z(x) = y(x)$, then $z(x + t) = y(x + t)$ where t is the time shift (Kahvec et al. 2001; Oppenheim and Schafer 1975). An estimator $z(x)$ is said to be scale-invariant when multiplication of all elements of the sample (x_1, \dots, x_n) by an arbitrary non-negative value β results in multiplication of the estimator by the same value ($\beta z(x)$). Table 1 summarizes the properties of introduced extreme value index estimators. As indicated, The Moment and Hill estimators are both scale invariant; however, they are not shift invariant. Hence, for improperly centered data, using these estimators may result in misleading estimates of γ (Aban and Meerschaert 2001). Contrary to

Table 1 Properties of the extreme value index estimators

EVI estimator	Scale invariant	Shift invariant	Restriction
Pickands	Yes	Yes	–
Hill	Yes	No	$\gamma > 0$
Moment	Yes	No	–
PWM	Yes	Yes	$\gamma < 1/2$
Likelihood	Yes	Yes	$\gamma > -1/2$
Two-step	Yes	Yes	–
Three-step	Yes	Yes	–

the moment and Pickands estimators which are consistent for all real values of γ , the Hill estimator is restricted to $\gamma > 0$.

The Probability Weighted Moment and likelihood estimators are both scale and shift invariant and are defined and consistent for $\gamma < 1/2$ and $\gamma > -1/2$, respectively. As previously mentioned, there is no general explicit formulation for the likelihood estimator and it may not even exist (Zhou 2008). This problem is addressed in the Two-Step estimator which is scale and shift invariant, and provides an approximation to the likelihood approach. The Three-Step procedure, similar to the Two-Step estimator, is scale and shift invariant (Zhou 2008).

As mentioned before, the likelihood estimator is not reliable for small sample sizes. Martins and Stedinger (2000) demonstrated this issue by estimating the parameter γ for a random sample ($n = 15$), generated from a GEV distribution with a defined γ . They showed that for such a small sample size, utilization of the maximum likelihood estimator may lead to erroneous estimates of the extreme value index. Hosking et al. (1985) showed that the PWM estimator performs better than the maximum likelihood method in estimating upper quantiles of the GEV distribution when the sample size is small. Based on this reason, Hosking (1990) argued that the PWM estimator is superior to the maximum likelihood estimators.

Application of different estimators on real data may result in considerable discrepancies between estimated extreme value indices. Unfortunately, there is to-date no single estimator which has proved to be consistent and reliable under all circumstances. This means that the choice of the proper estimator is somewhat data-dependent. Nevertheless, there are a number of issues that are discussed in Section 6.

5 Rainfall Data

Daily long-term daily rain gauge time series data from 6 different locations across the globe namely, the United States, Australia, France and the Netherlands are used in this study. Figure 1 shows the location of gauges and Table 2 provides additional

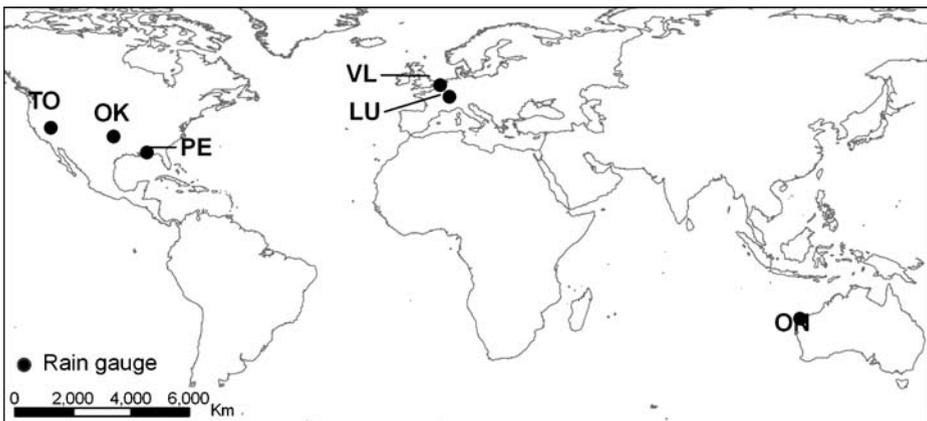


Fig. 1 Location of rain gauges used in this study (LU = Luxeuil; ON = Onslow; TO = Tonopah; OK = Oklahoma City; VL = Vlissingen; PE = Pensacola)

Table 2 Details of rainfall stations

Station name	Month-year	Latitude	Longitude
Onslow	01/1948–04/2006	−21.667	+115.117
Luxeuil	01/1946–10/2007	+47.800	+6.383
Vlissingen	01/1949–10/2007	+51.450	+3.600
Pensacola	02/1945–10/2007	+87.317	−87.317
Oklahoma City	12/1941–09/2007	+35.389	−97.600
Tonopah	10/1942–10/2007	+38.060	−117.087

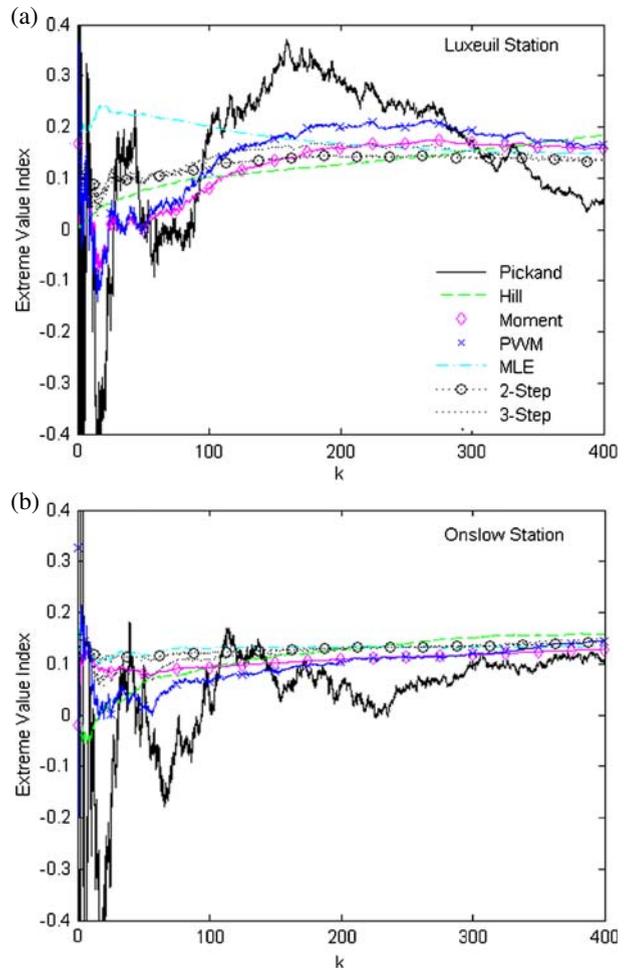
information such as station name, years of data, as well as the latitude and the longitude of the locations. The reason behind selecting various rain gauges from around the world is to address possible changes in the extreme value index with respect to general meteorological characteristics in future research.

Attention has been paid to the sample size such that the data for each rainfall station stretches over at least 38 (up to 71) years resulting in a sample size of approximately 14000 to 25000 data points for each station. It should also be noted that analyses are performed for each rainfall station separately. As mentioned in Section 2, extreme value analysis requires that the sequence of observations be independent and identically distributed. However, rainfall data are known to be autocorrelated. To overcome this issue, only the largest value from consecutive rainfall values that exceed a high threshold is used in the calculations. In order to do so, the clusters of dependent extreme values are to be identified first. Following Davison and Smith (1990), clusters of dependent extremes are defined when two occurrences exceeding the threshold are observed within three days of each other. Then, one extreme value is selected from each cluster to create a series of approximately independent values. This process is alternatively known as declustering. For a detailed review on different declustering methods, the interested reader is referred to Smith et al. (1997), Nadarajah (2001), Ferro and Segers (2003) and Fawcett and Walshaw (2007).

6 Results and Discussion

In this section, the extreme value index (EVI) is estimated for the rainfall data using the Pickands, Hill, Moment, Probability Weighted Moment (PWM), Likelihood (MLE), Two-Step and Three-Step estimators. Figure 2 presents the estimated values for Luxeuil and Onslow stations. It can be seen that the EVIs of Luxeuil and Onslow stations for large k (rank of the upper threshold) values are approximately 0.15 and 0.13 (ML estimator), respectively. It is worth recalling that a positive value of EVI indicates a heavy tail distribution. In both Fig. 2a and b, the Pickands approach (solid line) shows a large variability especially for small values of k , whereas the other estimators seem to be more consistent. This behavior of the Pickands estimator has also been reported in previous studies (e.g. Rosen and Weissman 1996). One can argue that using this estimator in practical applications may result in misleading estimates of the extreme value index, and thus the extreme quantiles. As shown in Fig. 2a and b, the Hill and Pickands estimators give the largest and smallest estimates of the extreme value index. The figures also indicate that the Two-Step approach, which is an approximation to the likelihood estimator tends to provide reasonable

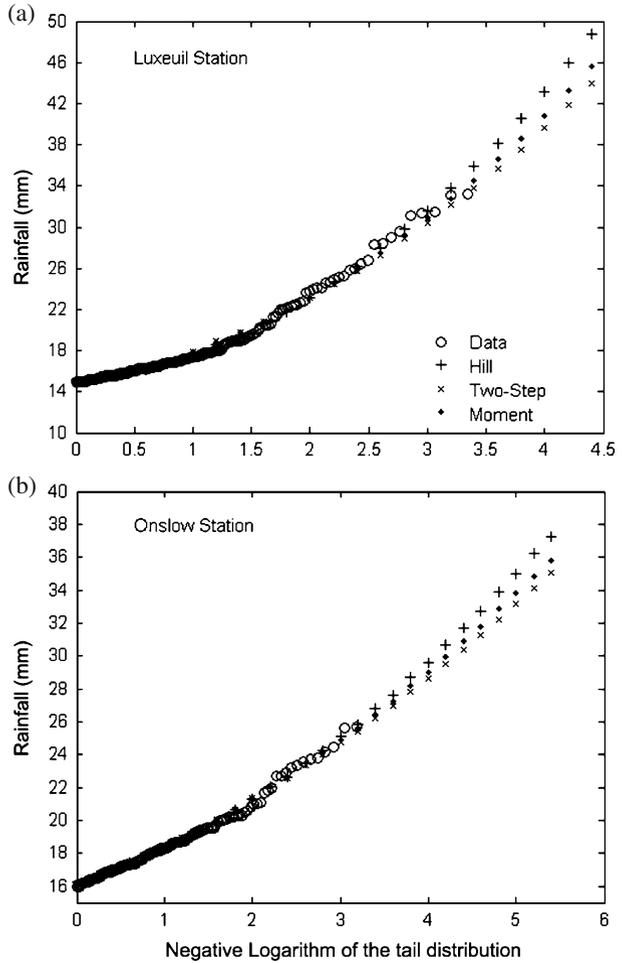
Fig. 2 Estimated EVIs for Luxeuil and Onslow stations: γ_P (solid line), γ_H (dashed line), γ_M (diamond mark), γ_{PWM} (x-mark), γ_{MLE} (dashdot line), γ_{2P} (o-mark), γ_{MLE} (dotted line) (a, b)



estimates often having a smoother trend. In both figures, the Two-Step (o-marked) and Three-Step (dotted line) are almost identical. This is due to the fact that the Three-Step approach follows the same mathematical formulation as that of the Two-Step with one more iteration in order to obtain the extreme value index.

Having estimated the GEV parameters, one can estimate the occurrence probability of extreme rainfall values. Figure 3a and b display the so-called exponential plot (Chambers 1977) for Luxeuil and Onslow stations. More precisely, the graphs show observed and estimated (extrapolated based on the fitted GEV distribution) rainfall data versus the negative logarithm of the tail distribution ($-\ln(1 - \Psi(x))$). Practically, this plot can be used to determine the occurrence probability of rainfall above a certain threshold ($1/e^x$). The graph is known as the exponential plot because it converges to a straight line when γ approaches zero, resembling exponential behavior (Bernardara et al. 2008). For lower-bounded (heavy tail) distributions

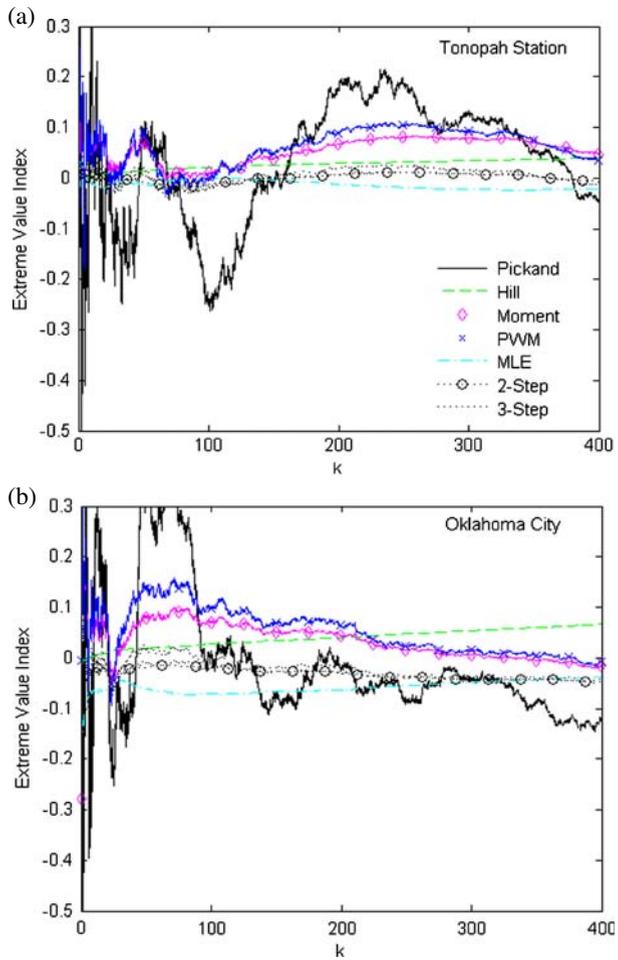
Fig. 3 Extrapolation of the extreme value distribution for Luxeuil and Onslow stations using the hill (*plus mark*), moment (*x-mark*) and two-step (*dotted line*) estimators (**a, b**)



($\gamma > 0$), the exponential plot grows faster than a straight line and bends toward up at larger values. In Fig. 3a and b, the increasing trend of the graphs reflect heavy tail behavior for the underlying probability distributions. In the figures, the estimated rainfall values, obtained based on the Hill (plus-marked), Tow-Step (x-marked) and Moment (dotted line) estimators are presented. The Hill approach seems to follow the trend of larger values, while the Two-Step method models the entire data set. The results of the Maximum Likelihood and Three-Steps are not included as they are very similar to the Two-Step estimator.

Figure 4a and b present γ versus k values for Tonopah and Oklahoma City stations, respectively. In both figures, the EVIs tend to zero for large values of k that suggest weaker tails than those of Luxeuil and Onslow (see Fig. 2). Figure 5a and b display the exponential plot of the observe and estimated rainfall data for Tonopah and Oklahoma City stations. As shown, the graphs converge to straight lines. This

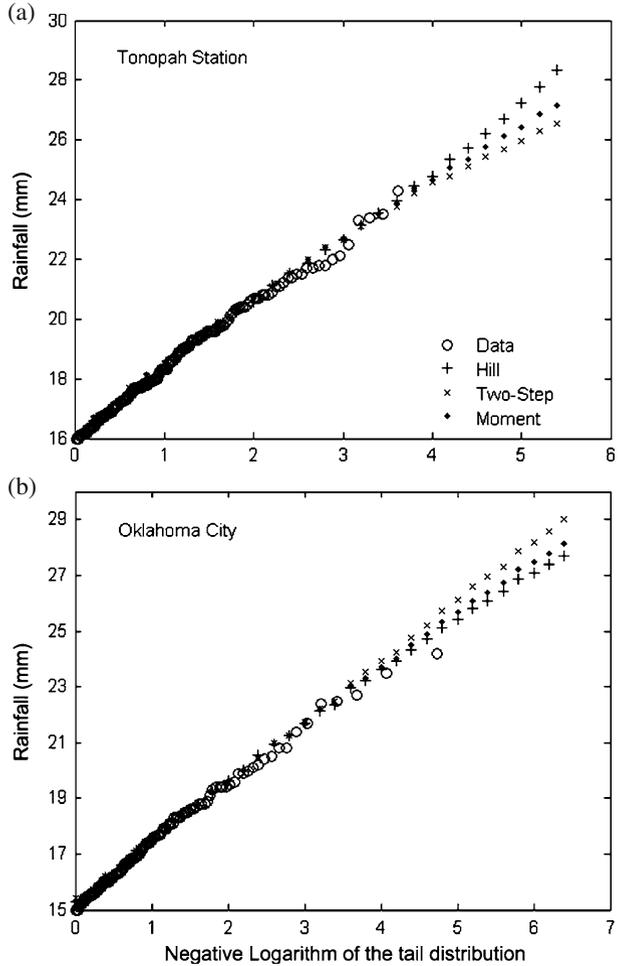
Fig. 4 Estimated EVIs for Oklahoma City and Tonopah stations: γ_P (solid line), γ_H (dashed line), γ_M (diamond mark), γ_{PWM} (x-mark), γ_{MLE} (dashdot line), γ_{2P} (o-mark), γ_{MLE} (dotted line) (a, b)



indicates that the fitted GEV distribution is unbounded. Recall that in the limits, as γ approaches zero the GEV distribution is unbounded. The graphs show that when the extreme value index approaches zero, the occurrence probability of rainfall above a certain threshold increases almost linearly with respect to rain rate.

Figure 6a and b plot the extreme value index versus k for Vlissingen and Pensacola stations, respectively. Estimated extreme value indices for large k values are $\gamma = -0.07$ (ML estimator) for Vlissingen and $\gamma = -0.08$ (ML estimator) for the Pensacola station. A negative value for γ suggests that the distribution has a finite right end-point. Notice that the Hill estimator always yields a positive γ even if the distribution has a right end-point. In such case the Hill estimator must be discarded. Extrapolation of the fitted distribution functions for Vlissingen and Pensacola stations are presented in Fig. 7a and b. As shown, for negative values of γ , the estimated rainfall values grow slower than a straight line and bend toward down at larger values. It is noted that in Figs. 3, 5 and 7, the estimated rainfall values are

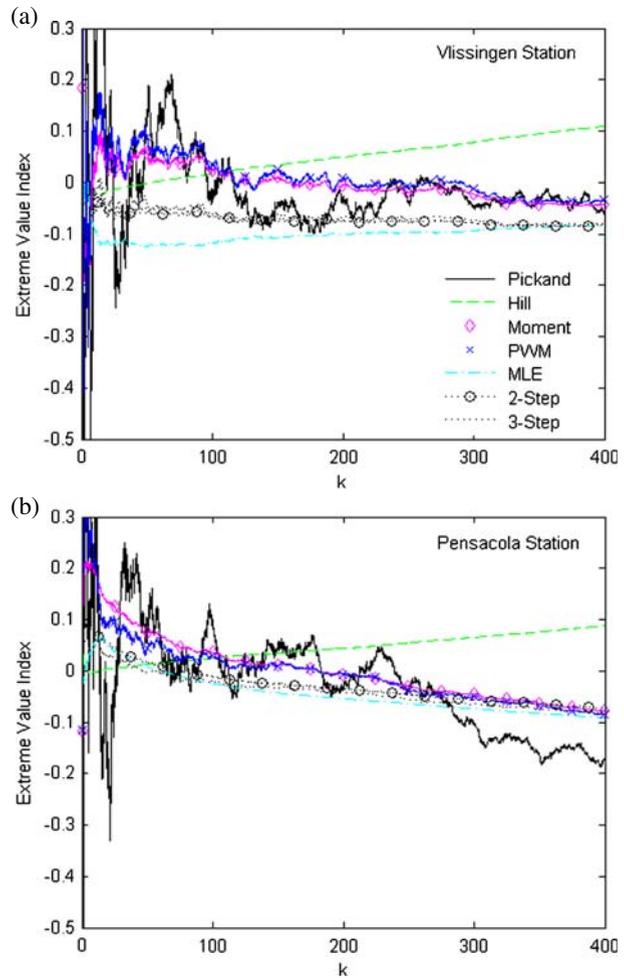
Fig. 5 Extrapolation of the extreme value distribution for Oklahoma City and Tonopah stations using the hill (*plus mark*), moment (*x-mark*) and two-step (*dotted line*) estimators (**a, b**)



based on the k values that correspond to the 98th percentile of the observations in each station.

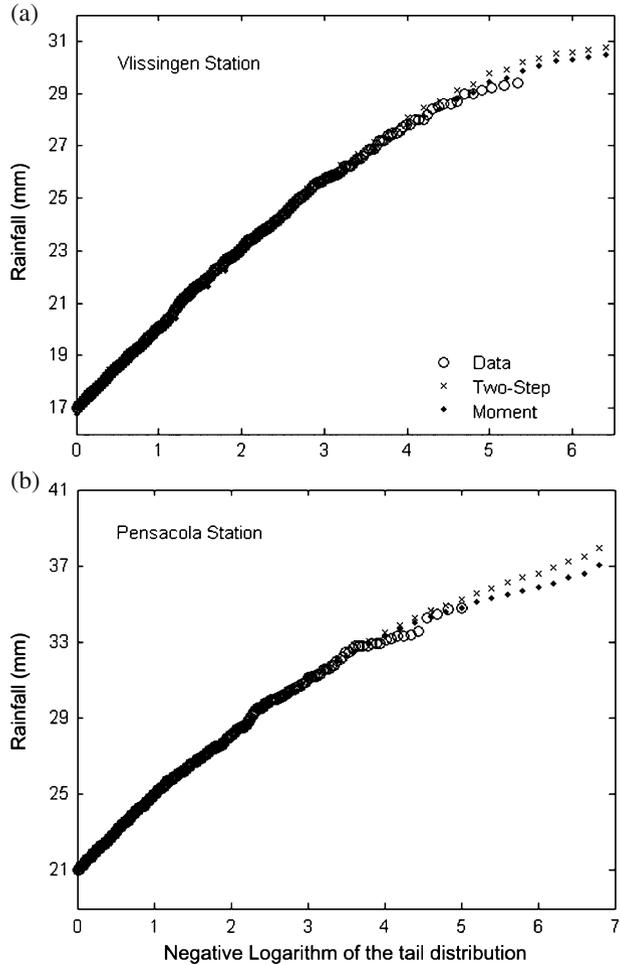
In practical applications, an appropriate estimator may be selected according to goodness-of-fit tests or other statistical tools rather than on theoretical considerations. Therefore, an important step in the process of estimator identification is to test whether the resulting model fits the observations. Table 3 lists the root mean squared error (RMSE) values for different estimators based on the 98th percentile of the observations as the threshold in each station. In all cases, the likelihood and Two-Step estimators resulted in the least RMSE values. One can see that there are considerable differences between the RMSE values of different estimators except for Onslow station where the Moment, PWP, MLE and Two-Step estimators were found to be equally good. Furthermore, the table clearly indicates that the Pickands estimator is inferior to the other estimators.

Fig. 6 Estimated EVIs for Vlissingen and Pensacola stations: γ_P (solid line), γ_H (dashed line), γ_M (diamond mark), γ_{PWM} (x-mark), γ_{MLE} (dashdot line), γ_{2P} (o-mark), γ_{MLE} (dotted line) (a, b)



To evaluate the uncertainty of the extreme value index estimators, discussed in this study, the bootstrap technique (Efron 1979; Davison and Hinkley 1997; Carpenter and Bithell 2000) is performed as follows. For each rainfall station, 1000 subsets of the original dataset is randomly selected such that each randomly selected subset contains 25% of the original sample size. The process is repeated for 50% and 75% of the original dataset. For each randomly selected sub-sample, the extreme value index is estimated using the estimators discussed in this paper. The mean absolute error (MAE) of the estimated extremal indices obtained from the random sub-samples are then calculated with respect to the extreme value index obtained from the original dataset. The mean absolute error which is a quantity often used to measure how close simulations (here, estimates from the random sub-samples) are to the observations (here, estimates from the original dataset) is defined as: $MAE = \frac{1}{n} \sum_{i=1}^n |\gamma_{Si} - \gamma_{Oi}|$, where γ_{Si} and γ_{Oi} represent extremal indices from simulated and observed datasets, and n is the number of pairs (Cox and Hinkley 1974;

Fig. 7 Extrapolation of the extreme value distribution for Vlissingen and Pensacola stations using the moment (*x-mark*) and two-step (*dotted line*) estimators (**a, b**)



Bernardo and Smith 2000). Table 4 summarizes the MAE values of the estimated extreme value indices for Luxeuil, Tonopah and Vlissingen stations. One can see that for the largest sample size (75%), the ML estimator led to the least mean absolute error (compare columns 4, 7 and 10 in Table 4). For the smallest sample size (25%), on the other hand, the Two-Step estimator for Luxeuil and Vlissingen stations

Table 3 RMSE of the fitted models (LU = Luxeuil; ON = Onslow; TO = Tonopah; OK = Oklahoma City; VL = Vlissingen; PE = Pensacola)

EVI estimator	LU	ON	TO	OK	VL	PE
Pickands	0.64	0.36	0.53	0.68	0.39	0.72
Hill	0.24	0.38	0.30	0.61	N/A	N/A
Moment	0.29	0.21	0.35	0.33	0.28	0.38
PWM	0.31	0.21	0.32	0.31	0.27	0.38
Likelihood	0.22	0.21	0.29	0.27	0.19	0.36
Two-step	0.20	0.21	0.22	0.27	0.19	0.38

Table 4 MAE values of the estimated extreme value indices from different estimators (LU = Luxeuil; TO = Tonopah; VL = Vlissingen)

EVI estimator	LU			TO			VL		
	25%	50%	75%	25%	50%	75%	25%	50%	75%
Pickands	0.129	0.112	0.103	0.110	0.108	0.063	0.110	0.099	0.048
Hill	0.077	0.037	0.022	0.054	0.036	0.026	N/A	N/A	N/A
Moment	0.067	0.048	0.035	0.051	0.054	0.022	0.073	0.060	0.024
PWM	0.066	0.060	0.028	0.064	0.056	0.019	0.069	0.055	0.020
Likelihood	0.090	0.040	0.020	0.033	0.020	0.018	0.051	0.011	0.007
Two-step	0.062	0.051	0.024	0.040	0.024	0.019	0.043	0.014	0.008

and the ML estimator for Tonopah station resulted in the least uncertainty in the estimated values. Overall, the table indicates that the ML and Two-Step estimators are subject to less variability.

7 Summary and Conclusions

The primary goal of this research was to investigate different extreme value index estimators as they are important for assessing risk for extreme rainfall events and thus floods. Different parametric and semi-parametric EVI estimators were employed in this study. The results showed that the semi-parametric Pickands approach was subject to large variability especially for small values of k while the other estimators seemed to be more consistent. Additionally, Pickands estimated the smallest extreme value index in most cases whereas Hill estimator provided the largest estimates of EVI.

The maximum likelihood, Two-Step and Three-Step methods were generally similar as they follow similar mathematical formulation. The Two-Step approach, known as an approximation to the likelihood estimator, provided robust estimates of the extreme value index for a wide range of k . The results shown in Figs. 2, 4 and 6 indicated that the outputs of the Two-Step and Three-Step were very similar although the Tree-Step takes an additional iteration in order to improve the accuracy. The Moment, PWM and ML estimators were robust at large k values but often inconsistent at small k -s. Overall, the parametric Two-Step and ML estimators were preferred over the semi-parametric Pickands and Hill estimators due to their robustness and consistency. The Two-Step was also given priority over the other parametric estimators for its stable performance for different values of k .

Attention must be paid to estimator characteristics and restrictions before any practical application. The Moment and Hill estimators are both scale invariant; however, they are not shift invariant. Contrary to the moment and Pickands estimators which are consistent for all real values of γ , the Hill estimator is restricted to $\gamma > 0$. The Probability Weighted Moment and likelihood estimators are both scale and shift invariant, and they are defined and consistent for $\gamma < 1/2$ and $\gamma > -1/2$, respectively.

Having estimated the extreme value index using different estimators, the distribution function of extreme values was extrapolated using Generalized Extreme Value Distribution. Such extrapolation can be employed to estimate occurrence probability of rainfall above a certain threshold. In cases where γ found to be positive, the Hill

approach seemed to follow the trend of larger values whereas the Two-Step method tended to model the entire data set. In other words, a few very large values at the tail may affect the Hill estimates more than the Two-Step estimates. In Fig. 5a, for example, one can see that the Hill estimator gives larger values, perhaps because of the upper few points that bend toward up. On the other hand, in Fig. 5b, where the upper few points bend toward down, the Hill estimator gives smaller values than the Two-Step estimator. However, this cannot be simply generalized and more in-depth research is required to evaluate the sensitivity of the EVI estimators to the extremes and outliers.

The uncertainty of the extreme value models is addressed using the bootstrap technique. Randomly selected subsets (1000 realizations) of the original dataset are used to estimate the extreme value index. Different sample sizes of 25%, 50% and 75% of the original dataset were used to evaluate the uncertainty with respect to the sample size. The mean absolute error (MAE) of the estimated extremal indices, estimated for the random sub-samples, are then calculated and compared to the extreme value index obtained from the original dataset. The results showed that, the ML and Two-Step estimators led to the least uncertainty in the estimated extremal indices for different sub-samples. On the other hand, the Pickands estimator exhibited the highest variability in the estimated extremal indices.

The results of this paper confirmed that different estimators may result in considerable differences in extreme value index estimates. Obviously, occurrence probability of extreme events as well as return level estimations strongly depend on γ and thus the choice of extreme value index estimator. While the choice of estimator may be somewhat data dependent, the results of this study showed that the parametric estimators are superior to the semi-parametric ones. In particular, the ML and Two-Step estimators seem to be more robust and consistent for practical application.

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