Modeling Radar Rainfall Estimation Uncertainties: Random Error Model

A. AghaKouchak[1]; E. Habib[2]; and A. Bárdossy[3]

Abstract: Precipitation is a major input in hydrological models. Radar rainfall data compared with rain gauge measurements provide higher spatial and temporal resolutions. However, radar data obtained form reflectivity patterns are subject to various errors such as errors in reflectivity-rainfall (*Z-R*) relationships, variation in vertical profile of reflectivity, and spatial and temporal sampling among others. Characterization of such uncertainties in radar data and their effects on hydrologic simulations is a challenging issue. The superposition of random error of different sources is one of the main factors in uncertainty of radar estimates. One way to express these uncertainties is to stochastically generate random error fields and impose them on radar measurements in order to obtain an ensemble of radar rainfall estimates. In the present study, radar uncertainty is included in the *Z-R* relationship whereby radar estimates are perturbed with two error components: purely random error and an error component that is proportional to the magnitude of rainfall rates. Parameters of the model are estimated using the maximum likelihood method in order to account for heteroscedasticity in radar rainfall error estimates. An example implementation of this approached is presented to demonstrate the model performance. The results confirm that the model performs reasonably well in generating an ensemble of radar rainfall fields with similar stochastic characteristics and correlation structure to that of unperturbed radar estimates.

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Introduction

Precipitation is an important input in many hydrological and climatological applications. Detailed information on surface precipitation at fine spatial and temporal scales is critically needed for driving land-surface hydrology models, for validation of numerical weather prediction and climate models, and for a variety of hydrologic prediction and forecasting studies. Hydrological modeling over large scales requires high resolution rainfall data to capture the temporal and spatial variability of rainfall, which has been proven to affect the quality of hydrological predictions (Goodrich et al. 1995; Shah et al. 1996; Faures et al. 1995; Hamlin 1983; Troutman 1983; Corradini and Singh 1985; Obled et al. 1994; Seliga et al. 1992; Dawdy and Bergman 1969; Rodda 1967). Traditionally, rain gauges have been the main source of surface rainfall measurements. However, rain gauges are sparsely available in space and suffer from lack of rainfall areal representation, which can be quite limiting for hydrologic applications. In recent years, the development of weather radar systems has provided rainfall information at higher temporal and spatial resolutions than was previously possible from rain gauge measurements. However, radar estimates are associated with several different error types that arise from various factors such as: beam overshooting, partial beam filling, and nonuniformity in vertical profiles of reflectivity, inappropriate *Z-R* relationship, spatial sampling pattern, hardware calibration, and random sampling error. (Rico-Ramirez et al. 2007; Hunter 1996; Austin 1987; Wilson and Brandes 1979). Measurement biases resulting from inappropriate *Z-R* relationship and subgrid scale variability of rainfall, among others, are also sources of discrepancy between radar estimates and rain gauge measurements (Jordan et al. 2003; Ciach et al. 2007). Quantification and characterization of such uncertainties and their effects on hydrologic applications has proved to be a challenging task (Krajewski and Smith 2002). It is expected that uncertainties in rainfall input data will propagate into predictions from hydrologic models (Morin et al. 2005; Hossain et al. 2004; Borga 2002; Sharif et al. 2002; Winchell et al. 1998; Vieux and Bedient 1998; Pessoa et al. 1993); therefore, accurate characterization of radar rainfall errors and the induced uncertainties in hydrological modeling is extremely important.

One way to assess radar rainfall uncertainties is to simulate an ensemble of possible radar fields stochastically. A reasonable ensemble of radar rainfall data is a representation of possible uncertainties in radar rainfall estimates. Stochastically generated rainfall ensembles are used as input to hydrological models to assess model uncertainties. There are a number of methods for simulation of multivariate rainfall fields. Wilks (1998) introduced a Markovian stochastic model for simulation of daily precipitation, using mixed exponential distribution for simulation of non-

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zero amounts. Following the Wilks approach, Brissette (2007) presented an algorithm to improve the efficiency of the model when data are contaminated with noise. Fowler et al. (2005) used a semi-Markov approach for simulation of multisite monthly rainfall data. Kim et al. (2008) applied the Markov chain method to account for temporal dependencies and the direct acyclic graph method to describe spatial dependencies of daily rain gauge data. One limitation of multisite rainfall simulation is that accounting for spatial and temporal dependencies simultaneously is not straightforward. Pegram and Clothier (2001) introduced the String of Bead model for radar rainfall simulation based on two autoregressive time series, one at the image scale and the other at the pixel scale, assuming log-normal marginal distribution for radar rainfall at the pixel scale. Seed and Srikanthan (1999) modeled space-time radar rainfall data using a multifractal (multiplicative cascade) approach, where each level of the multiplicative cascade was linked in time using a different ARMA(1,1) model.

An alternative approach to obtain an ensemble of rainfall fields is to simulate error fields and impose them over observed rainfall data. Krajewski and Georgakakos (1985) introduced a two-dimensional nonstationary random field generator for daily radar error. The application of this model in hydrological modeling is limited due to its low temporal resolution. Germann et al. (2006) proposed that the radar estimates can be perturbed with stochastic fields in order to obtain an ensemble of radar estimates. In a recent study, Ciach et al. (2007) developed an operational approach based on empirical investigations of joint samples of radar and ground surface data whereby the radar rainfall uncertainties consist of a systematic distortion function and a stochastic component. The radar-error components are then estimated using nonparametric estimation schemes. Based on the work presented by Ciach et al. (2007), Villarini et al. (2009) developed a radar rainfall filed generator and a model to produce probability of exceeding maps of radar rainfall conditioned on given radar rainfall estimates.

The aim of this paper is to introduce a radar rainfall uncertainty tool that is rather simple to implement, yet it offers some realistic features of the radar rainfall error. The presented model is fairly simple in its implementation such that it can be applied for practical hydrologic-driven applications. Furthermore, the model can be applied for spatial (e.g., a watershed) and temporal (e.g., event-by-event) localized applications. In this study, we generate an ensemble of possible radar fields by perturbing radar estimates with two error terms: a purely random component and an error component that is proportional to the magnitude of rainfall rates. The sum of both error terms is then used to perturb the radar estimates. The error terms are included in the *Z*-*R* relationship and are represented with two model parameters. The exponent and multiplicative factor of the *Z*-*R* relationship are estimated simultaneously with the error model parameters using the maximum likelihood model in order to account for heteroscedasticity in radar rainfall error estimates. Heteroscedasticity, alternatively known as variance heterogeneity, occurs when the distribution of residuals is dependent on the indicator variable (Petersen-Øverleir 2004; Proietti 1998; Wang et al. 2005).

An example implementation of this approach over a midsize watershed is presented to demonstrate the model performance. In order to obtain estimates of the radar rainfall error across the study area, ground reference rainfall data are obtained from high resolution rain gauge measurements over the watershed. The differences between reference surface rainfall data and radar estimates are considered and termed as observed error. In the provided example, an ensemble of radar rainfall data is simulated by perturbing the radar estimates with generated error terms. Subsequent runs of a hydrological model using simulated realizations of radar rainfall fields would then allow an assessment of uncertainty propagation due to the precipitation input.

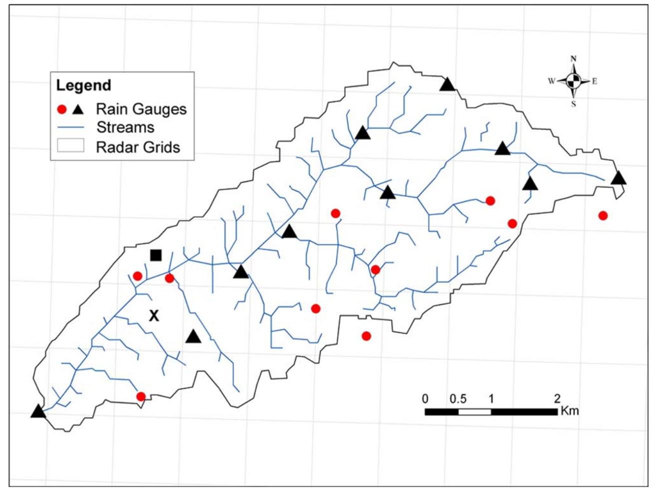

Fig. 1. Goodwin Creek experimental watershed, Mississippi, USA. The watershed is equipped with a dense network of rain gauges (marked with circle and triangle symbols) from which, ten rain gauges marked with triangle symbols are used for model validation.

The paper is organized as follows. After the introduction, the study area and available data are briefly introduced. The next section discusses the model and methodology. Then, the parameter estimation is described in detail. An example is then presented to demonstrate the model performance. Spatial and temporal dependencies of error fields as well as simulated radar rainfall fields are also investigated. The last section summarizes the results and conclusions.

Study Area and Data Description

The Goodwin Creek experimental watershed located in the north central part of the state of Mississippi, USA was selected for this study (Alonso and Binger 2000). The area of the watershed is approximately 21 km^2. Its average annual rainfall, measured at the climatological station near the center of the watershed, is 1,440 mm, and the mean annual runoff is approximately 144 mm at the watershed outlet. The topographic elevation ranges from 71 m at the outlet to 128 m at the watershed divide with reference to mean see level.

The main motivation to select the Goodwin Creek experimental watershed is that it is very well gauged and one can obtain a reasonable representation of radar error and its temporal and spatial dependencies by comparing radar estimates and rain gauge measurements. The radar data used in this study are the Level II reflectivity (*Z*) data collected at the Memphis National Weather Service WSR-88D (Crum and Alberty 1993) operational radar site (KLCH site). The radar station is located approximately 110 km to the north of the watershed. Radar data are available in polar coordinates at a resolution of $1° \times 1$ km for different elevation angles starting at 0.5° and with a time sampling interval of 5 to 6 min. Since the interest is in surface rainfall information, we use data from the lowest elevation angle of the radar beam and interpolated the polar reflectivity data onto a regular 1 km $\times$ 1 km Cartesian grid (Fig. 1). The Goodwin Creek watershed is equipped

Table 1. Summary Statistics of Rainfall Accumulations for Each Storm

Event	Date, time	Duration (h)	Mean (mm)	Standard deviation
Event 1	4/8/02 20:30	5:00	18.3	4.6
Event 2	5/2/02 22:30	4:30	27.6	4.8
Event 3	5/3/02 7:30	6:00	46.8	3.8
Event 4	9/26/02 14:00	10:00	13.5	5.9

with a dense network of 31 rain gauges with temporal resolution of 15 min. The data is collected by the National Sediment Laboratory of the U.S. Department of Agriculture in Oxford, Mississippi and is available to the public. Rain gauge measurements of four storms recorded on April, June and September, 2002 over the watershed is selected for this study. However, during this period only 20 rain gauges were in operation. Fig. 1 shows the position of gauges and radar pixels in the watershed and Table 1 displays summery statistics of lumped rainfall accumulations for each storm.

Methodology

As mentioned previously, radar measurements are associated with different error types (Wilson and Brandes 1979; Austin 1987; Hunter 1996; Pegram and Clothier 2003; Rico-Ramirez et al. 2007) that can be mainly grouped into three classes: (1) physical biases; (2) inaccuracies in *Z-R* relationship; and (3) random error (Jordan et al. 2003). Physical biases such as ground clutters, beam broadening, beam blockage, etc. are not of our concern in this paper and can be removed using different algorithms as described in Steiner and Smith (2002), Venkatesan et al. (1997), and Dixon et al. (2005). Random error, however, is the most difficult type of radar error and will remain even after removing the bias with an appropriate *Z-R* function. The superposition of random errors from different sources is one of the main factors in uncertainty of radar estimates. Ciach et al. (2007) and Villarini et al. (2009) suggested that the systematic error has to be removed before any analysis on the radar random error component. Similarly, the model presented here does not intend to capture all types of radar error especially those related to physical issues such as beam blockage that may result in no rain at parts of a radar image.

One way to simulate multiple realizations of radar rainfall fields is to perturb radar estimates with random error fields. Recent studies (e.g., Ciach et al. 2007; Habib et al. 2008) indicated that the radar error may depend on the magnitude of the rain rate. In order to account for such dependence, the following formulation is suggested:

$$R = \left(\frac{Z}{A}\right)^{1/b} + \left(\frac{Z}{A}\right)^{1/b}\varepsilon_1 + \varepsilon_2 \tag{1}$$

where R=rain rate; Z=reflectivity; ε_1=proportion error; ε_2=purely random error; A=multiplicative factor in *Z-R* relationship; and b=exponent in *Z-R* relationship.

In this error formulation, R represents rainfall ground reference measurements which are typically obtained from rain gauge observations. The term $(Z/A)^{1/b}\varepsilon_1$ represents the error component that is proportional to the magnitude of reflectivity or rain rate, whereas ε_2 accounts for purely random error sources. Assuming normally distributed error, the parameters of ε_1 and ε_2 are mean and SD. Ciach et al. (2007) showed that unbiased radar rainfall random error can be reasonably described by a normal distribution (in a multiplicative error model). This assumption of a Gaussian distribution of error is reasonable due to the fact that the overall radar error is a sum of errors from different sources, as previously explained. Based on the central limit theorem, the sum of all errors will tend toward a Gaussian distribution, regardless of the distribution of each individual source of error. In general, the model structure allows for the selection of an appropriate probability distribution for radar error (not necessarily Gaussian) based on available data. However, using the likelihood function for parameter estimation may become computationally too expensive. Note that the aim of this study is to present a simple model that performs reasonably well with minimum computational costs, even though this may result in a nonperfect agreement in some cases.

The error terms in Eq. (1) can be summed up to one error term representing the total error $[\varepsilon=(Z/A)^{1/b}\varepsilon_1+\varepsilon_2]$

$$R_i - \left(\frac{Z}{A}\right)^{1/b} \rightarrow \varepsilon(0,\sigma^2) \tag{2}$$

where

$$\sigma^2 = \sqrt{\left[\mathrm{std}\left(\frac{Z_i}{A}\right)^{1/b}\right]^2 \sigma_1^2 + \sigma_2^2} \tag{3}$$

The term σ is the SD of total error (both error terms) in Eq. (1). Substituting probability density function of normal distribution [Eq. (4)] into Eq. (1) with some algebra yields the following likelihood function [see Eq. (5)]:

$$\varepsilon(Y_1, \ldots, Y_n|0,\sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\left(-\frac{\sum_{i=1}^{n} Y_i^2}{2\sigma^2}\right) \tag{4}$$

where:

$$Y_i = R_i - \left(\frac{Z_i}{A}\right)^{1/b}$$

$$L = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\left\{\left[\mathrm{std}\left(\frac{Z_i}{A}\right)^{1/b} \cdot \sigma_1\right]^2 + \sigma_2^2\right\} - \frac{1}{2}\frac{\sum_{i=1}^{n}\left[R_i - \left(\frac{Z_i}{A}\right)^{1/b}\right]^2}{\left[\mathrm{std}\left(\frac{Z_i}{A}\right)^{1/b} . \sigma_1\right]^2 + \sigma_2^2} \tag{5}$$

where L=likelihood function; σ_1=SD of the proportion error; and σ_2=SD of the random error.

In Eq. (4), the term Y refers to the total error associated with radar data defined as rain gauge measurements (R_i) minus radar estimates $[(Z/A)^{1/b}]$.

Parameter Estimation

The model parameters b, A, σ_1^2, and σ_2^2 are to be estimated simultaneously using the maximum likelihood method whereby the parameters are determined such that the likelihood function (probability of the sample data) is maximized. From a practical viewpoint, the method of maximum likelihood has proven robust and reliable in a number of hydrologic applications (see Sorooshian et al. 1983; Haddad et al. 2007; Montopoli and Marzano 2007). For details on the parameter estimation techniques using

Table 2. Model Parameters Estimated for Each Storm

Event	A	b	σ_1^2	σ_2^2
Event 1	43	1.8	0.18	0.37
Event 2	125	1.3	0.12	0.40
Event 3	95	1.3	0.11	0.50
Event 4	119	1.45	0.05	0.36

the maximum likelihood, readers are referred to Brandt (1999). Using this method, reflectivity is transformed into rain rate with parameters that describe the uncertainty of radar estimates.

For all rainfall events used in this study, the model parameters are estimated using Eq. (5) and the maximum likelihood method. The estimated model parameters are listed in Table 2. Note that the parameter estimation procedure must be controlled to make sure that the bias of estimated rain rates is negligible. This can be achieved by allowing for a minimal bias ($\pm$ 2%) and estimating the parameters such that the likelihood function is maximized and the overall bias is kept within the minimally allowed bias value.

Simulation of Radar-Error Fields

Using stochastic simulation techniques, one can generate multiple equally probable (likely) realizations with similar statistical properties. The process is termed as conditional simulation when generated fields are conditioned on some observed (input) data. In the present study, radar-error fields are simulated conditioned on observed error values estimated by comparing rain gauge measurements and radar estimates at rain gauge locations.

Simulation procedure used in this study can be summarized in five steps: First, using the estimated parameters (A and b) obtained form Eq. (5), reflectivity fields are transformed into radar estimates. Second, an $m \times n$ normally distributed field termed as ε_2 is simulated using the model parameter σ_2, where m and n are the dimensions of the reflectivity field. In the third step, ε_1 is simulated using the model parameter σ_1 and multiplied by the radar estimates field obtained in the first step. In order to condition on observed error values, at rain gauge locations a random value form a normal distribution with the SD of σ_1 is selected such that the sum of both error terms is similar to the observed error value. At pixels where no rain gauges and thus no observed error values are available, ε_1 is randomly selected from the distribution. Fourth, both simulated error fields are imposed on the radar estimates field in order to obtain one realization of simulated radar estimates. In the last step, the second to fourth steps are repeated to obtain an ensemble of radar estimates.

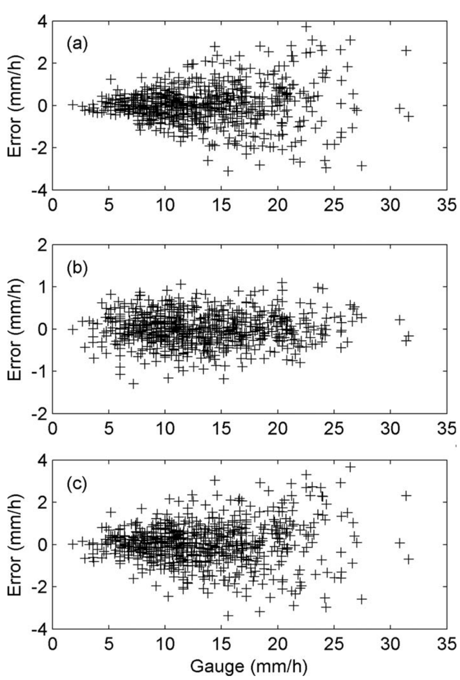

Fig. 2. (a) Simulated proportion error; (b) simulated random error; and (c) total error versus the rain gauge measurements

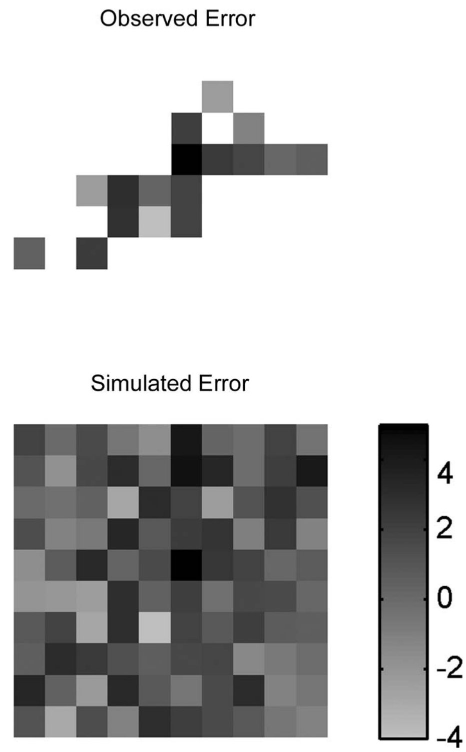

Fig. 3. (Top): Observed error values; (bottom): one conditionally simulated radar-error field

In the following example, the error model, introduced above, is employed to generate an ensemble of radar rainfall error fields. Figs. 2(a and b) demonstrate simulated proportion error $[(Z/A)^{1/b}\varepsilon_1]$ and random error (ε_2) versus the rain gauge measurements for Event 4. As shown, ε_1 is proportional to the magnitude of rain rate whereas random error remains independent. Fig. 2(c) presents the total error which is the sum of both error terms.

Following the simulation steps as described above, an ensemble of radar estimates was generated based on Eq. (1). For Event 4, Fig. 3 (top and bottom) show one observed error (differences between radar estimates and gauge measurements) and one conditionally simulated error field, which itself is the sum two error terms introduced previously. One can see that in the simulated realization, observed error values are honored at their locations. Simulated error fields are then imposed on radar estimates in order to obtain multiple realizations of radar estimates. As an example, Fig. 4 displays an unperturbed radar field on Sept. 26, 2002 at 18:00 GTM and one realization of simulated radar field after perturbation with the error.

Figs. 5(a–d) plot 500 realizations of radar rainfall estimates

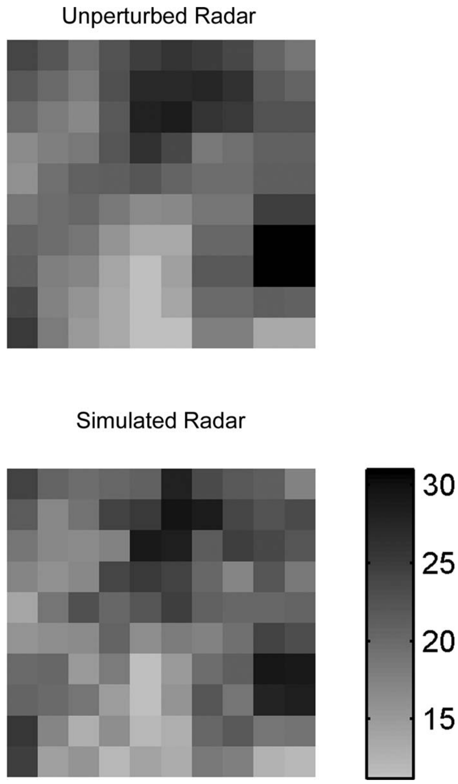

Fig. 4. (Top): Unperturbed radar field; (bottom): one realization of simulated radar field

over the x-marked pixel (Fig. 1) for the selected storms. The dashed line represents unperturbed radar estimates $[(Z/A)^{1/b}]$, whereas the gray lines are perturbed radar realizations [summation of three terms on the right side of Eq. (1)] over the entire

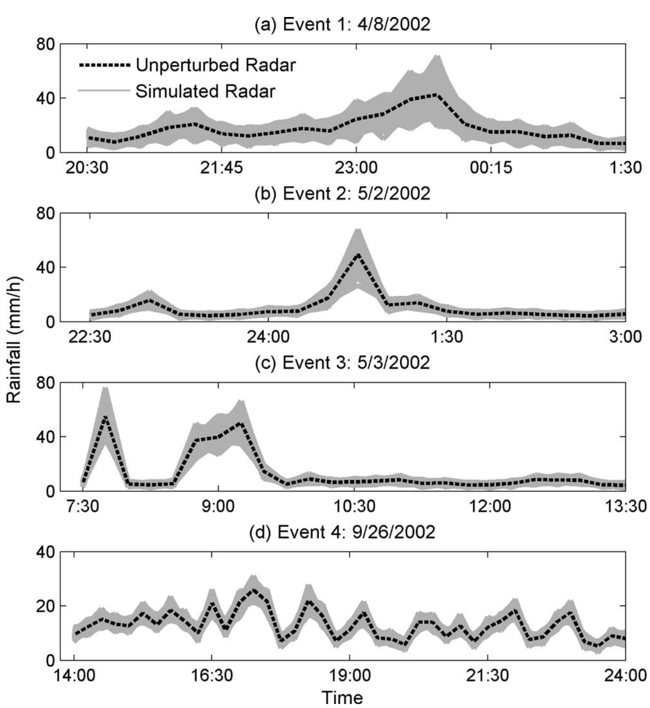

Fig. 5. 500 realizations of simulated radar data over the x-marked pixel (Fig. 1) for the selected storms

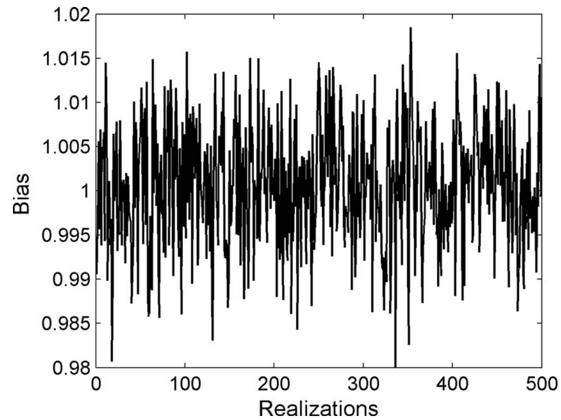

Fig. 6. Bias of 500 simulations over the x-marked pixel (Fig. 1) for Event 4

storm. Note that this formulation grants unbiased simulated realizations with respect to the total rainfall accumulations over the period of interest. Fig. 6 demonstrates the bias over the x-marked pixel for 500 simulations, whereby the bias is defined as the sum of the simulated radar data divided by the sum of unperturbed radar estimates. One can see that the bias for all realizations is desirably around the minimal bias allowed in parameter estimation.

In order to validate the model, ten rain gauge stations are removed (Fig. 1) and the model parameters are estimated based on the remaining data. Table 3 presents the model parameters after removing ten rain gauge stations. Figs. 7(a–d), respectively, show ensembles of radar rainfall estimates (500 realizations) for the selected storms over the square-marked pixel after removing ten rain gauge stations. The solid and dashed lines represent rain gauge measurements and unperturbed radar estimates, respectively. The gray lines are simulated radar realizations over the entire storm. One can see that the estimated uncertainty associated with radar data encloses rain gauge measurements except at few time steps. It is noted that the estimated uncertainty is described by uncertainty parameters (σ_1 and σ_2) that depend on the agreement between radar estimates and gauge data. In fact, large uncertainty parameters, and thus, high variability in simulated fields is expected when the deviation of rain gauge measurements from radar estimates is significant and vice versa. In Fig. 7(a) where at some intervals the deviation of gauge measurements from radar estimates is significant, the estimated uncertainty is wider than in Fig. 7(d). In Fig. 7(d), although the estimated uncertainty is smaller than Fig. 7(a), the gauge measurements are reasonably enclosed.

In the following analysis, the model performance is further evaluated in terms of the distribution function of observed and simulated error after removing ten rain gauge stations. Figs. 8(a–d) demonstrate the empirical cumulative distribution func-

Table 3. Model Parameters Estimated for Each Storm after Removing Ten Rain Gauge Stations

Event	A	b	σ_1^2	σ_2^2
Event 1	50	1.75	0.21	0.62
Event 2	217	1.35	0.10	0.59
Event 3	209	1.30	0.14	0.55
Event 4	99	1.45	0.06	0.40

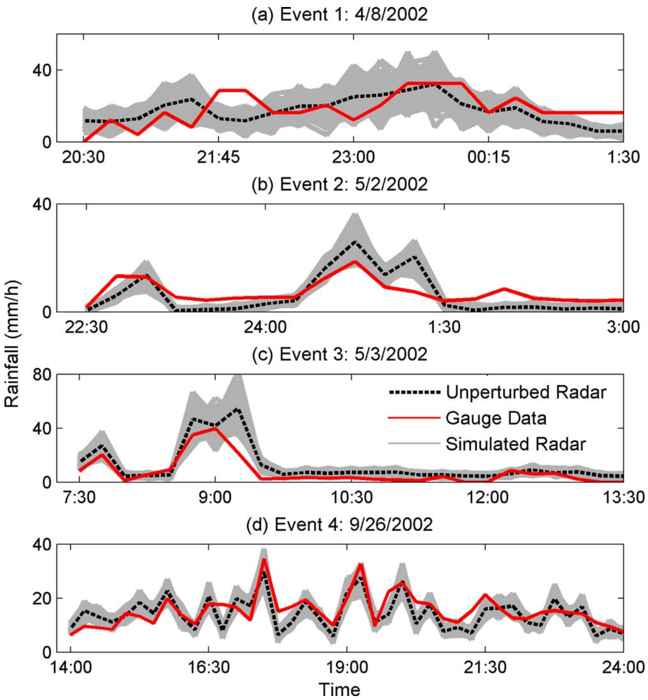

Fig. 7. 500 realizations of simulated radar data over the square-marked pixel (Fig. 1)

tions (CDFs) of the observed radar error and the 500 simulations of radar-error data for Events 1–4, respectively. The black solid line represents CDF of observed error values, whereas the gray lines are CDFs of simulated radar error. The dashed lines are theoretical normal CDFs obtained with the mean and SD of observed error values. As shown, the CDFs of simulated radar error

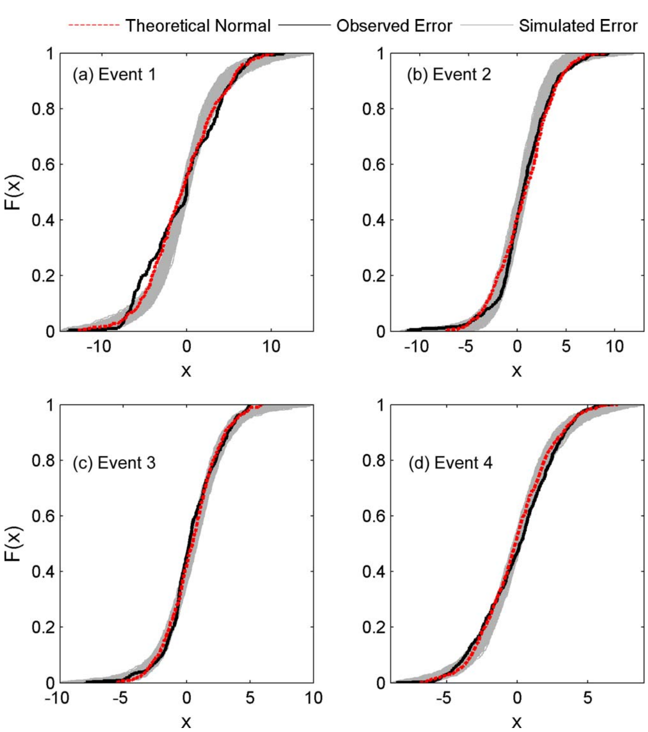

Fig. 8. Empirical CDFs of the observed (black lines) and 500 simulated radar-error data (gray lines) for Events 1 to 4

enclose those of the observed data except for Fig. 8(a). While in Figs. 8(b–d) the distribution of radar error can be described with normal distribution, in Fig. 8(a) the deviation of radar error from normality (the difference between the black solid and dashed lines) is quite considerable. It is also noted that the model slightly overestimates the upper/lower tail of the error values. This occurs specifically when the distribution of observed radar error is not perfectly symmetrical. For example, in Fig. 8(d), observed error values fall within −8.4–6.1 (see the black solid line in Fig. 8) while simulated values range between approximately −8.5–8.5 (gray lines in Fig. 8).

Spatial Dependencies

Krajewski and Georgakakos (1985) and Krajewski et al. (1993) proposed space-time stochastic modeling of the radar rainfall error. However, due to unavailability of true representation of rainfall in space and time, most studies were based on simplified assumptions regarding the spatiotemporal dependencies of the radar error and its distribution (Carpenter and Georgakakos 2004; Butts et al. 2004). Smith et al. (1989) modeled random error assuming no temporal or spatial correlation beyond the size of one radar pixel (4 km × 4 km). However, significant spatiotemporal dependency may exist in radar-error fields (Jordan et al. 2003). Carpenter and Georgakakos (2004) recognized this issue and indicated the need to account for error dependencies. Jordan et al. (2003) introduced a random cascade model to generate error fields, taking into account error correlations. The results showed non-negligible dependencies in the error field especially in space. Ciach et al. (2007) showed that the radar random error component was correlated in space and time and the estimated correlations were higher in the cold season. Habib et al. (2008) showed that the spatial dependence of radar error is not negligible and can have a significant effect on streamflow hydrologic predictions. Villarini et al. (2009) employed the modified exponential function to describe the spatial and temporal dependencies of radar random error. They reported that the modified exponential function captured the spatial and temporal structure of their empirical results. In addition, a number of models have been developed to generate spatially correlated two-dimensional random fields using covariance or a variogram model (see Journel and Huijbregts 1978). However, Germann et al. (2006) argued that using a simple variogram model or a covariance matrix may lead to unrealistic radar fields. In the present study, the purely random error term (ε_2) is defined to be uncorrelated in space. In the proportion error term, however, when ε_1 is multiplied by the radar estimates [Eq. (1)], the resulting error field $[(Z/A)^{1/b}\varepsilon_1]$ will be spatially correlated due to the fact that underlying radar field is correlated. In other words, even without explicitly assuming a spatial correlation for (ε_1), the product of an uncorrelated error field (ε_1) and a correlated field (unperturbed radar estimates) will be spatially correlated. In this study we do not intend to simulate rainfall fields with the same exact spatial dependence structure; however, the general dependence structure of simulated radar fields should be similar to the dependence structure of the unperturbed radar field. The reason for this is that if some neighboring pixels show high dependence in the unperturbed image, the same pixels in the simulated fields should also be similar in terms of their general dependence structure.

In the present study, the Kendall's τ rank correlation matrix is used to assess the dependence structure of the simulated radar fields after imposing the two error components. The Rank corre-

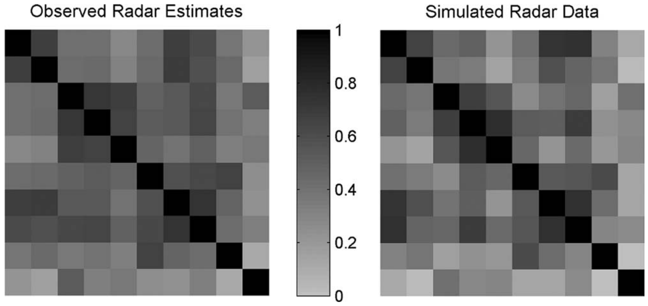

Fig. 9. Kendall correlation matrices of (left): unperturbed radar estimates; (right): simulated radar data

lation matrix, unlike Pearson's linear correlation matrix, evaluates the degree of association in terms of ranks which is known as concordance. Using a rank correlation such as Spearman's or Kendall's correlation, one can describe the correlation structure of data independent of their marginal distribution. For n radar pixels a $n \times n$ Kendall correlation matrix can be derived as

$$\tau_{ij} = \binom{n}{2}^{-1} \sum \sum_{u<v} \operatorname{sign}(x_u^i - x_v^i) \cdot \operatorname{sign}(x_u^j - x_v^i) \tag{6}$$

where i,j=rows and columns in Kendall matrix and u,v=data-point numbers.

As an example, Fig. 9 (panel) shows the Kendall correlation matrix of the 10-h-long storm in Sept. 2002 (Event 4) calculated for the unperturbed fields $[(Z_i/A)^{1/b}]$. For simplicity, only ten radar pixels located within the watershed are considered. The graph shows a 10×10 symmetrical matrix that describes the rank correlation between pairs of radar pixels. Fig. 9 (right panel) demonstrates the correlation matrix of the simulated radar fields after perturbing with both of the proportion and random error fields. As shown, the rank correlation matrix of simulated radar fields is not considerably different than that of unperturbed radar field and the overall dependence structure is preserved mainly due to the fact that the underlying correlation of unperturbed radar estimates is dominant. Similar results were obtained for the other rainfall events.

Temporal Dependencies

In a recent work over the same study area, Habib et al. (2008) reported that the temporal autocorrelations of the total error (estimated as the ratio between gauge and radar estimates) were rather low at the first time lag and close to zero for larger time lags. For this reason, the error terms, ε_1 and ε_2, were assumed to be temporally uncorrelated in the formulation of the model. However, similar to the case of spatial dependence, when ε_1 is multiplied by the radar estimates, the resulting error field $[(Z/A)^{1/b}\varepsilon_1]$ will carry some of the temporal self-correlation of the unperturbed radar field. Fig. 10 compares the temporal autocorrelation of the unperturbed fields of Event 4 to those of the 500 simulated radar fields for the square-marked pixel in Fig. 1. It is noted that the simulated fields have slightly weaker autocorrelations due to perturbation. However, the trend of temporal autocorrelations is similar to that of unperturbed field indicating that the random errors added to the unperturbed radar estimates did not destroy the underlying temporal autocorrelation. That means even without

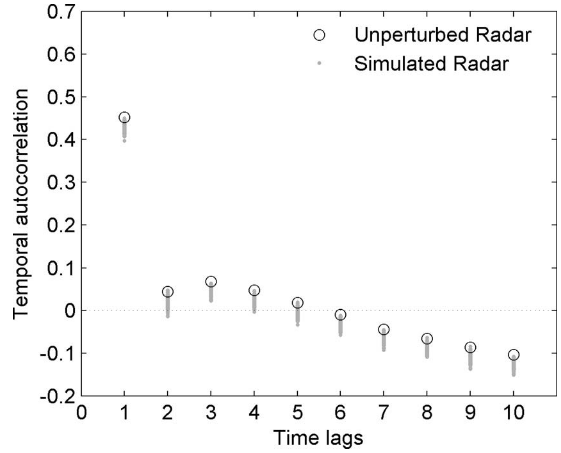

Fig. 10. Temporal autocorrelation of unperturbed radar estimates and 500 simulated radar realizations for the square-marked pixel

explicitly accounting for temporal dependencies in the error model explained above, the simulated radar have realistic temporal self-correlation characteristics.

Summary and Conclusions

It is recognized that total radar rainfall estimation error is a result of different error sources that are superposed on one another. The simulation model introduced in this paper uses this concept to generate an ensemble of radar rainfall fields through perturbing radar estimates with conditionally simulated random error fields. The resulting realizations of radar rainfall fields can be used for assessing the impact of uncertainties associated with radar rainfall estimates on hydrologic predictions. In this model, radar uncertainty is included in the *Z-R* relationship whereby radar estimates are perturbed with two error components: purely random error and an error component proportion to the magnitude of the rain rate. Introducing the multiplicative error term (ε_1) in the model is to account for the proportionality of rainfall error to the magnitude of rain rate reported in various publications (Habib et al. 2008; Ciach et al. 2007). In fact, one can intuitively argue that both error terms are random; one multiplicative and one additive: $[(Z/A)^{1/b}(1+\varepsilon_1)+\varepsilon_2]$, where the terms $1+\varepsilon_1$ and ε_2 are the multiplicative and additive error components, respectively.

Radar reflectivity fields are used as the input of the model to generate an ensemble of rainfall estimates. The *Z-R* relationship and model parameters are estimated simultaneously using the maximum likelihood technique in order to avoid heteroscedasticity in radar rainfall error estimates. As an example, application of this model for simulation of a radar ensemble was presented. The analyses were performed using LEVEL II reflectivity radar data obtained from the WSR-88D Memphis radar station. A dense network of rain gauge measurements was used to estimate model parameters. Using the presented model, multiple realizations of radar-error fields were generated and used to perturb radar estimates. The results showed that simulated realizations had similar statistical properties as those of unperturbed radar estimates and that the simulated ensemble encompassed ground reference measurements across the study area.

In order to validate the model, ten rain gauge stations were removed and the model parameters were estimated based on the remaining data. The results showed that the estimated uncertainties associated with radar data enclose rain gauge measurements with the exception of a few time steps where the deviation of gauge measurements from radar estimates was significant. It is worth remembering that the uncertainty parameters σ_1 and σ_2 dominate the variability of simulated radar fields. One can conclude that the estimated uncertainty depends on the agreement between radar estimates and rain gauge measurements such that the better the agreement, the smaller the estimated uncertainty and vice versa.

It should be noted that no explicit accounting for error autocorrelation was included in the model (both error terms, ε_1 and ε_2, were assumed to have no self-correlation in space and time). However, the self-correlation of the radar field will be carried forward when ε_1 is multiplied by the radar estimates resulting in the proportion error term $[(Z/A)^{1/b}\varepsilon_1]$. Self dependencies of the simulated radar rainfall fields were then investigated by calculating both the temporal autocorrelation coefficients at various temporal lags, and the Kendall rank spatial correlation matrix. The results revealed that imposing radar-error fields over unperturbed radar estimates using the presented error model does not destroy the underlying spatial dependence structure. Furthermore, the temporal autocorrelations in the simulated fields were quite similar to that of the unperturbed radar field, which is mainly due to the fact that the temporal autocorrelations of the unperturbed radar field were dominant.

The problem of accounting for spatial and temporal dependencies of radar error warrants further investigations. In the provided example, the Kendall rank correlation matrix of simulated radar fields was not considerably different than that of unperturbed radar estimates; however, this may not be generalized. A reasonable estimation of spatial dependencies of ε_1 might be required to obtain simulated radar fields with a similar dependence structure as that of the unperturbed radar estimates.

Another simplification of our model is that the error components are assumed to be normally distributed. Such an assumption was tested and used by Ciach et al. (2007) and Villarini et al. (2009) in a multiplicative error model. However, the assumption requires further research to evaluate the appropriateness of this distribution for modeling lower and upper quantiles of radar error. The presented model was tested through investigation of the distribution functions of observed and simulated radar error. The results revealed that except for Event 1, the CDFs of the observed radar error were within the CDFs of the simulated radar error. It is noted that considerable deviation of radar-error distribution from normality may affect the model performance. While several studies showed that radar error with temporal resolution of hourly and above can be reasonably modeled using normal distribution (Ciach et al. 2007; Villarini et al. 2009), the distribution of high resolution radar-error data are still to be investigated. Theoretically, any probability distribution function can be fit and assumed for radar error based on available data. However, the parameter estimation based on likelihood approach may require extensive computational effort and time. We emphasize that in this study, the goal is to present a simple model that performs reasonably well with minimal computational costs, considering the fact that the simplifying assumptions behind the model may result in a nonperfect agreement in some cases.

The model is fairly simple in its implementation such that it can be simply applied for practical hydrologic-driven applications avoiding the too-simplistic models that are often used in hydrologic modeling analysis (e.g., models that assume pure random error with fixed variance and no dependence structure; Carpenter and Georgakakos 2004; Butts et al. 2004). Unlike recent elaborate radar-error models (e.g., Ciach et al. 2007) that are designed for large-sample and seasonal applications, the current model is geared towards spatial and temporal localized applications [i.e., for an event-by-event and for a particular spatial location (e.g., a watershed)]. Notice that when model parameters are estimated based on long-term data, a relatively similar uncertainty bound will be simulated for rainfall events. However, rainfall error and its variability may vary from event to event. The presented model can be tailored for analysis of event-by-event cases given there are enough samples for parameter estimation.

An interesting issue for future research is to examine the variability of model parameters with respect to different types of storms. One can see that in Table 2, the exponent of Event 1 is quite different than those of the other three events. This difference could be due to the characteristics of the corresponding rainfall regime. Such analyses require extensive high quality radar rainfall estimates and rain gauge measurements.

It is hoped that this error model can be used for generating reasonable radar ensembles, specifically for the purpose of investigating error propagation in modeling hydrologic processes. However, the results cannot be considered as general results. It is noted that the analyses presented here were performed on rainfall data with high temporal resolution (15 min), which may have some drawbacks (e.g., random errors in tipping-bucket measurements). On the other hand, aggregation of data in time may result in the loss of temporal variability. Further research including simulations over different spatial and temporal scales and different gauge network setups are required to verify the robustness of the approach. In this study, a large number of rain gauges, which may not be typically available, were used as a ground reference to estimate the parameters of the error terms. The sensitivity of the results to the number of gauges available in the model domain needs to be investigated in order to evaluate the effects of sampling error on model outputs. Another limitation on the implementation of this method is that unreliable surface gauge measurements may result in erroneous parameter estimation and thus create an unrealistic ensemble and inaccurate measures of uncertainty. Also, it is worth pointing out that we have used an individual gauge in a pixel to represent the true areal average rainfall over a pixel size, which may not be very accurate. However, based on the available data and the spatial resolution of radar data (1 km^2), this was the best possible approximation of the true areal average rainfall values. Current efforts are underway to investigate these various issues in order to evaluate the statistical robustness and transferability of the model to different temporal and spatial scales.

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