Multi-hazard scenarios for analysis of compound extreme events

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Key Points:

- We present a framework for multivariate analysis of natural hazards driven by multiple forcings
- Choice of marginal probability distribution and copula can significantly influence design and hazard scenarios
- Bayesian approach for parameter estimation illuminates the uncertainties of different multi-hazard scenarios

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Abstract

Compound extremes correspond to events with multiple concurrent or consecutive drivers (e.g., ocean and fluvial flooding, drought and heatwaves) leading to substantial impacts such as infrastructure failure. In many risk assessment and design applications, however, multi-hazard scenarios of extremes and compound events are ignored. In this paper, we review the existing multivariate design and hazard scenario concepts, and introduce a novel copula-based weighted average threshold scenario for an expected event with multiple drivers. The model can be used for obtaining multi-hazard design and risk assessment scenarios and their corresponding likelihoods. The proposed model offers uncertainty ranges of most likely compound hazards using Bayesian inference. We show that the uncertainty ranges of design quantiles might be large, and may differ significantly from one copula model to the other. We also demonstrate that the choice of marginal and copula functions may profoundly impact the multi-hazard design values. A robust analysis should account for these uncertainties within and between multivariate models that translate into multi-hazard design quantiles.

1 Introduction

The interdependence between two or more hazard drivers, which may not necessarily be extreme events individually, may trigger significant extreme impacts - a phenomenon known as a compound event [*Leonard et al.*, 2014; *Wahl et al.*, 2015; *Mehran et al.*, 2017; *Vahedifard et al.*, 2016]. Compound events (or impacts) may occur as a result of one of the following situations [*Field*, 2012]: 1. two or more simultaneous or successive extreme events (e.g., simultaneous extreme precipitation and storm surge, *Moftakhari et al.* [2017a]), 2. combinations of extreme events with underlying conditions that amplify the impact (e.g., droughts and heatwaves, *Mazdiyasni and AghaKouchak* [2015]), or 3. combinations of events that are not themselves extreme but collectively lead to an extreme event or impact (e.g., a moderate coastal flood occurring during above average tide, *Moftakhari et al.* [2015]).

Frequency and severity of compound events are expected to increase in the future [*Kopp et al.*, 2017; *Mechler and Bouwer*, 2015; *Field*, 2012], which in turn elevate their associated risks, as defined by combination of threatening events (a.k.a. hazards) and adverse consequences (e.g., exposure and vulnerability) [*Tessler et al.*, 2015]. This necessitates a deeper understanding of compound extremes and their impacts. Reliable and accurate characterization of compound hazards, as one important element of risk, requires in-depth research to advance the existing theoretical frameworks and tools [*Leonard et al.*, 2014].

In this study, we review the existing multivariate design and hazard scenario concepts, and introduce a new copula-based methodology that offers uncertainty ranges of the most likely compound hazards using Bayesian inference. Copulas have been proven to be a valuable tool to describe and analyze the dependence structure of multiple variables in hydrology and climatology [*De Michele and Salvadori*, 2003; *Favre et al.*, 2004; *Salvadori and De Michele*, 2004a,b; *De Michele et al.*, 2005], and have been employed as a vehicle to develop multi-hazard design scenarios [*Volpi and Fiori*, 2012; *Gräler et al.*, 2013; *Salvadori et al.*, 2014]. While uncertainty analysis has received a lot of attention in different branches of hydrology and climate science (*Sadegh and Vrugt* [2013, 2014]; *Sadegh et al.* [2015, 2018], and references therein), it is not broadly explored in multi-hazard design scenarios. This is specifically important given the relatively short length of our observations, which may translate into large uncertainty in the design/hazard scenarios [*Sadegh et al.*, 2017].

We also analyze the marginal and joint probability distributions of compound hazard events, and depict the importance of the choice of marginal distributions as well as copulas to model univariate and multivariate probabilities of natural hazards and compound events. Moreover, we propose a multivariate approach that estimates an expected hazard threshold level based on the weighted average of multiple critical levels/thresholds.

2 Methodology

2.1 Critical level for hazard assessment

In a typical hazard assessment problem, depending on the problem's dimension, the probability space is divided by an equal probability point, line, or surface, hereafter referred to as the critical layer. We define critical layer as [*Salvadori et al.*, 2011],

$$\boldsymbol{L}_q^P = \left\{ \boldsymbol{x} \in R^d \ : \ P(\boldsymbol{x}) = q \right\} \tag{1}$$

in which q is the critical probability level, $\boldsymbol{x}$ denotes a realization of the d-dimensional feasible space, and P is a d-dimensional probability distribution. In a univariate study, P reduces to a marginal distribution, while in a multivariate study, P is defined as a copula probability distribution that describes the correlation structure of the driving variables. $\boldsymbol{L}_q^P$ divides the feasible space into three sub-regions [*Corbella and Stretch*, 2012]:

- sub-critical (non-hazardous) region ($R_q^<$) which includes events (realizations) with probability, P, lower than the critical probability level ($P < q$);
- critical layer (e.g., point, line, or surface), $\boldsymbol{L}_q^P$, on which events hold equal probability, q;
- super-critical (hazardous) region ($R_q^>$) where events with probability, P, higher than the critical probability level, q, ($P > q$) fall.

Depending on the study goals, one might be interested in either sub- or super-critical (non-hazardous or hazardous) regions. This paper focuses on coastal flooding and hence, we focus on super-critical (hazardous) region, $R_q^>$ (i.e., high extreme values).

2.2 Marginal distributions

Estimating the critical level or return period of an extreme event typically involves fitting distribution functions. We use 17 different continuous marginal distribution functions to find a suitable model that optimally fits the available data. Distribution functions include 1. Beta, 2. Birnbaum-Saunders, 3. Exponential, 4. Extreme value, 5. Gamma, 6. Generalized extreme value, 7. Generalized Pareto, 8. Inverse Gaussian, 9. Logistic, 10. Log-logistic, 11. Lognormal, 12. Nakagami, 13. Normal, 14. Rayleigh, 15. Rician, 16. t location-scale, and 17. Weibull distributions (listed alphabetically). For a detailed description of these distributions refer to *Johnson et al.* [1993, 1994], and *Bowman and Azzalini* [1997].

The best marginal distribution is selected based on the Bayesian Information Criterion (BIC). The parameters of the marginal distributions are estimated through a maximum likelihood algorithm that minimizes the distance between empirical probability values and their modeled counterparts. A chi-square goodness-of-fit test is then employed to statistically examine whether or not data is sampled from the fitted distribution at 5% significance level [*Lewis and Burke*, 1949]. Visual comparison of the fitted distribution versus empirical probability values, as well as QQ plotting is also used to verify the acceptability of the distribution fit.

In many studies only one (or a few) marginal distribution(s) are used [*Zheng et al.*, 2015]. We argue that using a wide range of distributions is essential to minimize prior assumptions on the distributions of data by selecting the best fitted function. Any distribution holds some underlying assumptions, but our flexible approach strives to identify those closest to that of the underlying empirical distribution of data. We, however, share a common assumption with the literature that the underlying marginal distribution does not change over time [*Salvadori et al.*, 2014].

2.3 From univariate to multivariate multi-hazard analysis

Return period is a statistical measure of the expected recurrence interval of a hazard, such as flood, over an extended period of time. This statistical concept is frequently used for risk analysis and infrastructure design purposes. Univariate return period is defined as

$$\mathrm{RP}_q^1 = \frac{\mu}{\Pr\left(\boldsymbol{x} \in R_q^{>}\right)} = \frac{\mu}{1 - P_q^1}, \tag{2}$$

in which RP_q^1 represents the univariate return period, P_q^1 signifies the marginal probability at the critical probability level, q, and μ is defined as average inter-arrival time of observed events [*Salvadori et al.*, 2011, 2014].

The univariate return period concept can be extended to higher dimensions for multi-hazard analysis. However, in a multi-hazard case, it is important to consider the dependence between hazard drivers. Copulas have been widely used for modeling the dependence structure of two (or more) time-independent random variables, regardless of their marginal distributions [*Joe*, 2014; *De Michele and Salvadori*, 2003; *Favre et al.*, 2004; *Nelsen*, 2007; *Salvadori et al.*, 2007, 2014; *Grimaldi et al.*, 2016].

Nelsen [2003] informally defines 2-dimensional copulas as a mapping tool from $I^2(I \times I)$ space to I, in which $I \in [0, 1]$. $F_1(x_1) = \Pr(X_1 \leq x_1)$ and $F_2(x_2) = \Pr(X_2 \leq x_2)$ describe marginal distributions of continuous random variables X_1 and X_2, respectively, and $H(x_1, x_2) = \Pr(X_1 \leq x_1, X_2 \leq x_2)$ explains their joint probability distribution. Hence, $[F_1(x_1), F_2(x_2), H(x_1, x_2)]$ is a point in a 3-dimensional space I^3. According to Sklar's theorem [*Sklar*, 1959], there exists a copula function, C, for which $H(x_1, x_2) = C\,[F_1(x_1), F_2(x_2)]$. If marginal distributions, F_1 and F_2, are continuous, copula C is unique. Similarly, a copula can be constructed from a joint cumulative distribution, H, as $C(u_1, u_2) = H\left[F_1^{-1}(u_1), F_2^{-1}(u_2)\right]$ given $[u_1, u_2] = [F_1(x_1), F_2(x_2)]$. The definition of copula can similarly extend to d-variables,

$$C(\boldsymbol{u}) = H[F_1^{-1}(u_1), F_2^{-1}(u_2), \cdots, F_d^{-1}(u_d)], \tag{3}$$

where $\boldsymbol{u} = [u_1, u_2, \cdots, u_d]$.

We use the 26 bivariate copulas built into the Multivariate Copula Analysis Toolbox (MvCAT) [*Sadegh et al.*, 2017], which includes models with one to three degrees of freedom, namely 1. Gaussian, 2. t, 3. Clayton, 4. Frank, 5. Gumbel, 6. Independence, 7. Ali-Mikhail-Haq (AMH), 8. Joe, 9. Farlie-Gumbel-Morgenstern (FGM), 10. Gumbel-Barnett, 11. Plackett, 12. Cuadras-Auge, 13. Raftery, 14. Shih-Louis, 15. Linear-Spearman, 16. Cubic, 17. Burr, 18. Nelsen, 19. Galambos, 20. Marshall-Olkin, 21. Fischer-Hinzmann, 22. Roch-Alegre, 23. Fischer-Kock, 24. BB1, 25. BB5, and 26. Tawn copulas. The equations of these copulas are available in Table 1 of *Sadegh et al.* [2017].

MvCAT infers the copula parameters by tuning them to optimally fit the estimated joint probabilities to the associated empirical joint probabilities. This is a non-parametric approach for fitting copulas that uses pseudo-observations to find copula parameters. MvCAT includes two optimization and uncertainty analysis frameworks, namely a gradient-based local optimization and a hybrid-evolution Markov Chain Monte Carlo (MCMC) simulation [*Sadegh et al.*, 2017]. The local optimization option uses an "interior-point" algorithm [*Byrd et al.*, 2000; *Waltz et al.*, 2006], and performs a quick search of the feasible space at the expense of a small likelihood of getting trapped in local optima [*Sadegh et al.*, 2017]. MCMC, on the contrary, warrants finding an estimate of the global optimum at a small computational expense. The employed state-of-the-art MCMC algorithm numerically solves the Bayes' equation,

$$p(\boldsymbol{\theta}|\tilde{\mathbf{D}}) = \frac{p(\boldsymbol{\theta})p(\tilde{\mathbf{D}}|\boldsymbol{\theta})}{p(\tilde{\mathbf{D}})} \propto p(\boldsymbol{\theta})p(\tilde{\mathbf{D}}|\boldsymbol{\theta}), \tag{4}$$

to estimate posterior distribution of copula parameters $p(\boldsymbol{\theta}|\tilde{\mathbf{D}})$. In Equation 4, $\tilde{\mathbf{D}}$ denotes the empirical joint probability vector, $p(\boldsymbol{\theta})$ represents the prior distribution of copula parameters (uniform in our case), $p(\tilde{\mathbf{D}})$ is the evidence, and $p(\tilde{\mathbf{D}}|\boldsymbol{\theta})$ denotes the likelihood function. If we conveniently assume the error residuals (divergence between copula modeled, $d_i(\theta)$, and empirical joint probability values, $\tilde{d_i}$) are uncorrelated, Gaussian distributed with a zero mean and a constant variance (homosckedastic), the likelihood function could be defined as:

$$p(\tilde{\mathbf{D}}|\boldsymbol{\theta}) \cong \mathcal{L}(\boldsymbol{\theta}|\tilde{\mathbf{D}}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\tilde{\sigma}^2}} \exp\left\{-\frac{1}{2}\tilde{\sigma}^{-2}\left[\tilde{d_i} - d_i(\boldsymbol{\theta})\right]^2\right\}, \tag{5}$$

where, $\tilde{\sigma}$ is an estimate of the standard deviation of measurement error, which can be estimated on-the-fly in the MCMC simulation. Each of the posterior copula parameters can be used to derive the multi-hazard design scenarios. However, if uncertainty analysis is not desired, mode of the posterior distribution can be used for design purposes.

3 Multivariate hazard scenarios

Return period in a multivariate space, as an intuitive extension of its univariate case, is defined as [*Vandenberghe et al.*, 2011],

$$\mathrm{RP}_q^{2+} = \frac{\mu}{\Pr\left(\boldsymbol{x} \in R_q^{>}\right)} = \frac{1}{1 - C(\boldsymbol{u}^q)}. \tag{6}$$

in which $\boldsymbol{u}^q = F(\boldsymbol{x}^q)$, and $\boldsymbol{x}^q \in \boldsymbol{L}_q^P$. This approach follows the "OR" definition of the joint return period in section 3.3 of *Gräler et al.* [2013]. Also see *Salvadori et al.* [2016].

In this setting, an event with a pre-specified critical layer or joint return period can be selected for design purposes and/or hazard assessment. However, there are numerous combinations of $\boldsymbol{x}$ on the return period curve (associated with the critical layer) in a 2^+-dimensional problem, RP_q^{2+}, with equal probability [*Salvadori et al.*, 2011; *Volpi and Fiori*, 2012]. For example, there are infinite combinations of water level and fluvial discharge leading to statistically similar 100-year events, while their impacts can be drastically different. An intuitive approach to select among plausible pairs of $\boldsymbol{x}$ is to assign weights to them based on their associated copula density values [*Gräler et al.*, 2013; *Salvadori et al.*, 2011; *Corbella and Stretch*, 2012; *Salvadori et al.*, 2014; *Zheng et al.*, 2015]. The copula probability density function (pdf) is defined as [*Volpi and Fiori*, 2012],

$$h(\boldsymbol{x}) = \frac{\partial^d H(\boldsymbol{x})}{\partial x_1 \partial x_2 \cdots \partial x_d}. \tag{7}$$

We seek to offer an approach to select the desired points on the critical return period level, RP_q^{2+}, based on the copula density values and the underlying uncertainties. The critical return period level, RP_q^{2+}, is associated with the critical layer, $\boldsymbol{L}_q^P$. In the following, we review the literature on this topic, and introduce two novel concepts: i) uncertainty ranges of the most likely design scenario, and ii) an expected scenario derived from the weighted ensemble average of the most likely design scenarios.

3.1 Most likely scenario

The most common approach in selecting one design/hazard scenario, among feasible combinations with equal return periods, is to analyze the system under the most likely compound event. The most likely scenario coincides with the combination of hazards on the critical layer, $\boldsymbol{L}_q^P$, (a.k.a critical joint return period, RP_q^{2+}), with highest joint density level [*Salvadori et al.*, 2014] defined as,

$$\boldsymbol{x}^q = \operatorname{argmax}\ h(\boldsymbol{x}), \quad \boldsymbol{x} \in \boldsymbol{L}_q^P. \tag{8}$$

This most likely compound event, however, may not be the most severe among the possibilities in terms of impact. So, we further explore other possibilities to sample from the critical level, and evaluate their hazardousness.

3.2 Multiple samples on the critical layer, $\boldsymbol{L}_q^P$

To obtain a distribution of potentially hazardous combination of drivers, rather than one single combination as in Section 3.1, we draw weighted random samples of compound events from the critical layer, $\boldsymbol{L}_q^P$. The sampling approach uses the copula probability density function, $h(\boldsymbol{x})$, as weight [*Gräler et al.*, 2013]. The samples with higher joint probability density values have a higher chance of selection, but this method allows for perturbing a level of stochasticity into design scenarios and hazard assessment, consistent with the stochastic nature of hazard drivers. The output will be a set of forcing that can be used to run a numerical or conceptual model (e.g., hydrodynamic model of an estuary).

3.3 Uncertainty analysis of most likely scenario

The approach proposed in this paper helps describing uncertainties associated with statistical model structure by finding the copula family that best describes the correlation structure between two (or more) variables. It also helps quantifying uncertainties associated with parameter estimation procedure. This framework attributes modeling uncertainties to copula parameters, $\boldsymbol{\theta}$ [*Vrugt and Sadegh*, 2013], and estimates the most likely scenarios for each posterior copula parameter set (as opposed to the one scenario from best copula parameter; Section 3.1). This approach is mathematically defined as,

$$\forall \theta_i \in \boldsymbol{\theta} \qquad \boldsymbol{x}_i^q = \text{argmax } \ h(\boldsymbol{x}), \quad \boldsymbol{x}_i \in \boldsymbol{L}_q^P(\theta_i). \tag{9}$$

We treat each sample of the posterior distribution with equal weight, however, the most likely region of the posterior distribution inherently encompasses more samples. This method quantifies the uncertainties associated with the estimated most likely scenario, and analyzes the sensitivity of system to variation of this scenario.

Copula model parameters are prone to measurement and model structural errors. These errors preclude finding a "unique" parameter combination that is significantly better than others [*Vrugt et al.*, 2003]. Indeed, some parameters might be equally good according to a goodness-of-fit measure (the problem of "equifinality" [*Beven and Binley*, 1992]). Moreover, one parameter combination might be superior to others according to one goodness-of-fit index, and inferior based on another. Copula parameters also depend on the period of observation. It is hence suggested that a cohort of samples that are all acceptable provides more information about the system behavior as opposed to a best parameter combination, which is to be accepted as "true" representation of the system. Selecting best parameter may underestimate the uncertainties of the system [*Sadegh and Vrugt*, 2014]. See Supplementary Information, SI, for more detail.

3.4 Expected scenario

For design and risk assessment purposes, it is also useful to estimate an expected event that represents the non-extreme dynamics of the system. This reference scenario represents the situation which should be expected in any given year. This approach relaxes the need for defining a critical return period level, RP_q^{2+}. Following the concept of Cumulative Hazard proposed by *Moftakhari et al.* [2017b], a weighted average of most likely scenarios with return periods of 2, 10, 25, 50 and 100 years is calculated. Weights are then assigned according to the return period levels, and the approach is formulated as,

$$\begin{aligned} \boldsymbol{x}^{\mathrm{ex}} &= \left(\sum_{i=1}^{5} \frac{1}{\mathrm{RP}_{q_i}^{2+}} \ \operatorname{argmax} \ h(\boldsymbol{x}), \quad \boldsymbol{x} \in \boldsymbol{L}_{q_i}^{P}\right) / \left(\sum_{i=1}^{5} \frac{1}{\mathrm{RP}_{q_i}^{2+}}\right), \\ \mathrm{RP}_{\boldsymbol{q}}^{2+} &= [2, 10, 25, 50, 100]. \end{aligned} \tag{10}$$

This is a non-extreme threshold scenario and is not meant to replace extreme design scenarios. Weights used in Equation 10 are selected based on the most widely used return period levels. These weights are assigned as reciprocal of RP level, which are associated with their occurrence probability. Events with lower probability of occurrence (more extreme) yield higher design levels but get lower weights, and vice versa. We will now show how this approach can be implemented in a coastal flooding problem with two dependent flood drivers.

4 Results

In this study, we analyze compound flooding hazard (i.e., combined ocean and terrestrial flooding) in Washington, DC, USA. This area has a considerable number of infrastructure exposed to flooding in the Potomac River channel [*Ayyub et al.*, 2012; *Moftakhari et al.*, 2017b]. The dynamics of flooding is strongly determined by the nonlinear interactions between freshwater inflows and estuary water level [*Hoitink and Jay*, 2016] (See SI for detailed physical description of this interdependence). Thus, modeling the correlation structure between flood drivers is crucial for appropriate characterization of flooding dynamics. In fact, previous studies have shown that ignoring the interactions between ocean and terrestrial flooding can lead to biased risk estimates [*Moftakhari et al.*, 2017a]

Here, we consider the daily freshwater inflow estimates by the United States Geological Surveys (USGS; gauge number 01646500) and hourly water level observations provided by National Oceanic and Atmospheric Administration (NOAA; gauge number 8594900) as major flood drivers. The pair of interest for any given year is set to be the largest annual freshwater inflow to the estuary, and the corresponding largest observed hourly water level within ±1 day. We first investigate the inter-dependency of these two natural hazards in an 83-year record (hence 83 pairs of drivers) through different correlation coefficients namely, Kendall's rank ($r = 0.5274$, $p-$value $= 0.0000$), Spearman's rank correlation ($r = 0.7041$, $p-$value $= 0.0000$), and Pearson correlation coefficient ($r = 0.9125$, $p-\text{value} = 0.0000$), all of which display significant dependence between the two variables. Hence, we have used our proposed model to describe flood hazard considering both (terrestrial and ocean) drivers and their inter-dependencies.

We first select the best fitted marginal distributions, F_1 and F_2, to the observed flood drivers (Section 2.2) based on the BIC goodness-of-fit metric. Figures S2A and S2C (SI) show the fitted distributions (red line) compared to the observed (blue dots) river discharge and water level. Figures S2B and S2D display the QQ plots to visually examine the goodness-of-fit of the distributions to the observed data. In our study, a Log-logistic distribution is selected to fit both variables (Table S1). The Chi-square test for both drivers at 5% significance level also confirms our visual inspection that fitted distributions are acceptable.

Then, we evaluate 26 bivariate models using MvCAT toolbox [*Sadegh et al.*, 2017]. The copula parameters and their posterior distributions are inferred using MCMC simulation within a Bayesian framework. In this study, the Fischer-Hinzmann copula is selected as the best model to describe the dependence structure among the studied variables, based consistently on all goodness-of-fit criteria (AIC, BIC, Likelihood, NSE, and RMSE). However, this copula does not have a closed-form joint probability density function due to the "min" operator in its joint cumulative distribution function, which leaves the derivative undefined at $X_1 = X_2$ (when the two inputs to copula are equal). This impedes finding the most likely scenario (Section 3.1) of compound hazard effects, among others. Hence, we select the Joe

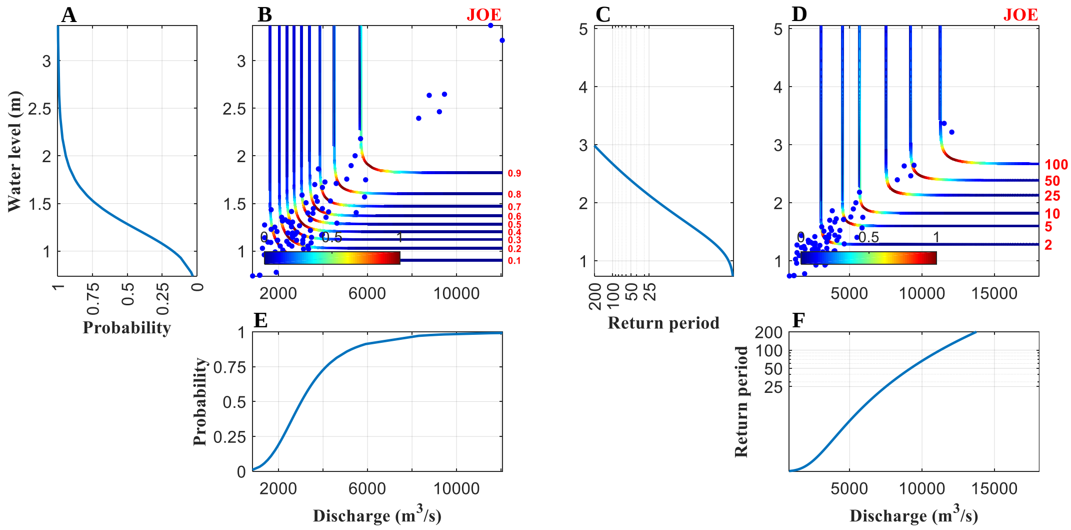

Figure 1. Marginal cumulative distributions of water level (A) and river discharge (E), and their associated univariate return periods (C and F; *y*-axis presented in log-scale). Joint probability isolines derived from Joe copula are displayed in panel B, and associated return period isolines are depicted in panel D. Both joint probability and multivariate return period isolines are color coded with joint density levels with blue representing lower densities and red denoting higher densities. Joint probability density values are re-normalized to [0,1] range for visualization proposes. Blue dots are observed pairs of river discharge annual maxima and associated water levels.

copula (second best model) for the rest of this study. Figure 1 shows the joint probability isolines (B) and return period levels (D) based on the Joe copula.

We then use the Joe copula model to derive design quantiles, and analyze the associated compounding hazards. Figure 2 shows the river discharge - water level fluctuation design pairs associated with a compound event with joint return period of 50-year based on the different approaches described in Section 3. The design values of river discharge and water level, based on the most likely design scenario are 9820.50 m^3/s and 2.48 m, respectively. These are larger than the design values derived through univariate analysis (river discharge of 9202.01 m^3/s and water level of 2.39 m). This clearly highlights that ignoring the interactions between flood drivers can lead to underestimation of the hazard.

To provide a broader range of multivariate design scenarios, Figure 2C shows 100 samples (black dots) randomly drawn from the $RP^2_{0.98}$ (50-year event) curve based on the weights assigned by the copula density values (Section 3.2). Design quantiles for this approach using the Joe copula range between 9202.00 and 12632.37 m^3/s for river discharge (a range that equals to 97% of the mean annual maximum river discharge) and 2.39 to 3.58 m for water level (a range that is as wide as 86% of average water level). The main limitation of this approach is that the ranges of discharge and water level are very wide and cover most of the entire distribution. Figure 2C also displays the uncertainty space of design quantiles based on the most likely scenario (Section 3.3). Each red dot represents the most likely scenario for one parameter set from the posterior distribution of the Joe copula, derived using MCMC simulation within a Bayesian framework. Each of these design levels could be acceptable given the available information in the observed data, which depicts the importance of taking

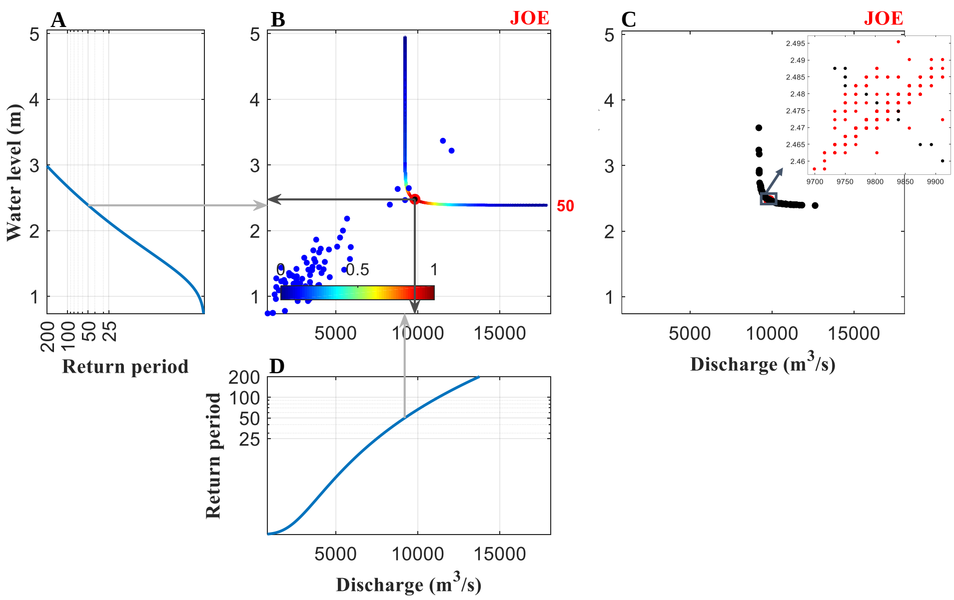

Figure 2. Multivariate design quantiles based on different approaches. Panel A and D display univariate return period curves for water level and river discharge, respectively. Y-axis for these two plots is in log-scale. Panel B shows a joint 50-year return period curve, color coded with the joint density levels. Light grey arrows display the univariate design quantiles, whereas dark grey arrows depict the most likely scenario design quantiles. Panel C displays, in black dots, multiple samples on the joint 50-year return period level randomly drawn with density levels as weight. In this plot, red dots show uncertainty space of most likely scenario design quantiles for posterior samples of the Joe copula model.

uncertainty quantification into consideration for an informed design and management practice.

Thus far, we have focused on the design quantiles and the associated hazards based on a pre-defined return period. Here, we propose a multivariate expected event regardless of return period level (see the theory in Section 3.4). Table S4 shows the expected events' boundary conditions for different copula families. In other words, the threshold levels of Table S4 represent the compound hazards of the system at any given year with the highest likelihood. For the Joe copula, an expected event is defined by a river discharge of 4492.23 m^3/s and a water level of 1.58 m. A closer look at Table S4 shows that the threshold quantiles of an expected event given different copula models is fairly constrained, and fall within the interval of [4490.77 m^3/s - 5042.82 m^3/s] for river discharge (a range that equals to 16% of the mean annual maximum discharge), and [1.58 m - 1.72 m] for water level (a range that equals to 10% of the mean water level). Note that expected scenario quantiles refer to a non-extreme event that is most likely to occur in any given year, and are significantly smaller than the extreme multi-hazard scenarios. Non-extreme scenarios show lower uncertainty ranges, compared to extremes; however, we design systems to withstand the extreme scenarios, and we should be wary of the uncertainties in our design and hence probability of failure.

One key question is that to what extent the choice of multivariate model (here, choice of copula) affects the estimated hazard. Figure 3 plots 100 weighted random samples on the $RP^2_{0.98}$ (50-year) curve (black dots) and uncertainty ranges of the most likely scenario based on the posterior distribution of copula parameters (red dots) for a group of six randomly selected representative copulas (also see SI). This figure shows that choice of copula model and

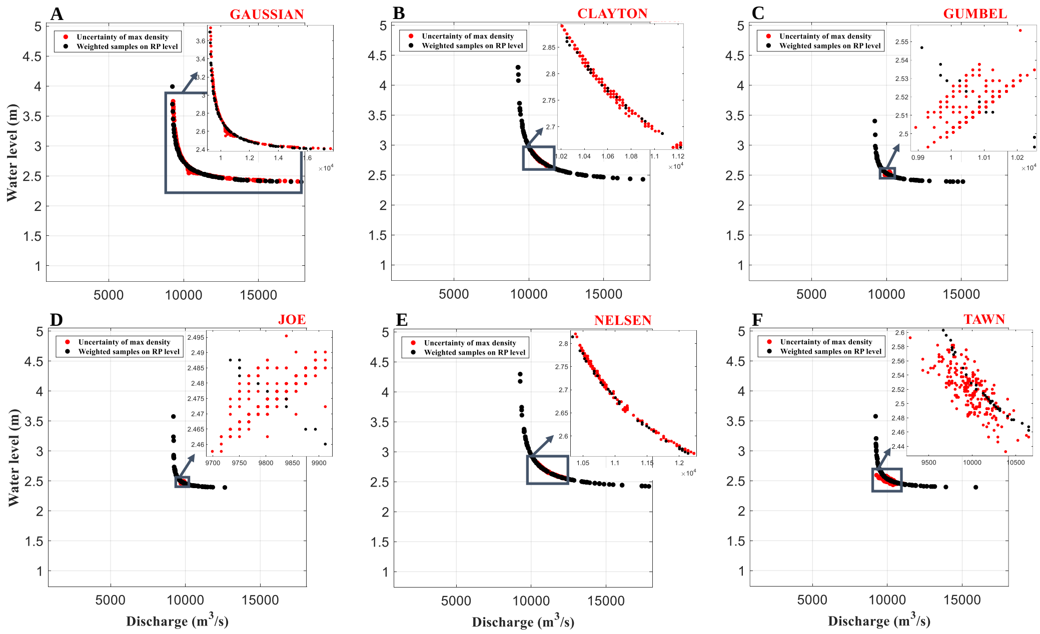

Figure 3. Multivariate design quantiles based on 100 weighted random samples on the critical layer (50-year joint return period curve) displayed with black dots, and uncertainty ranges of most likely scenario depicted with red dots for a set of six randomly chosen copulas, namely Gaussian (A), Clayton (B), Gumbel (C), Joe (D), Nelsen (E), and Tawn (F).

underlying uncertainty of copula parameters can potentially translate into large ranges of design (or critical layer) quantiles. The weighted random samples (black dots) on the $RP^2_{0.98}$ curve, for all the copulas, cover a relatively large interval as wide as 246% of the mean annual maximum discharge and 159% of the average water level. More importantly, the uncertainty ranges of the most likely scenario significantly differ from one copula model to another (Figure 3). For example, the uncertainty ranges of the most likely design quantile for the Joe copula model is as wide as 6% of the mean annual maximum discharge and 3% of the mean water level, respectively, while these ranges significantly expand for the Gaussian copula to 337% of the mean annual maximum discharge and 97% of the average water level, respectively.

Uncertainties in the most likely scenario (and other scenarios) stem from multiple sources, including the goodness-of-fit of the models (both marginal and multivariate model), model structural errors, posterior distributions of the copula model parameters, and even the observed joint probability errors. The observations to which the univariate and multivariate models are fitted to are often not long enough to sufficiently constrain the model parameters, specifically for multi-dimensional models [*Sadegh et al.*, 2017]. The length of record is also a constraining factor in terms of evaluating out-of-range return periods, requiring extrapolation which exponentially increases the design uncertainties. Currently, most publications in hydrology and climate journals consider very few multi-variate models in their analysis, which may lead to large biases and errors in estimated joint return periods, critical levels, etc. Such errors can be minimized through a rigorous copula and marginal fitting, chosen from a wide range of options. This ensures the selected copulas and marginals are good representatives of the under-study system. We also note that some copula families with rather similar performance metrics may show significantly different forms of probability isolines (see *Sadegh et al.* [2017]). This raises the question of which model should be trusted to singlehandedly provide the design quantiles. We demonstrate that a multi-model analysis provides more robust design quantiles and hence should be adopted by the community (Figure S1).

Finally, uncertainty estimates (as in Figure 3) should be transparently communicated to those responsible for infrastructure design and risk assessment, as well as to the public [*Covello et al.*, 1986; *Faulkner et al.*, 2007; *Adger et al.*, 2013; *Buchecker et al.*, 2013]. Neglecting uncertainties in the characterization of hazardous events can potentially lead to adopting inefficient or over-designed mitigation strategies [*Pielke et al.*, 2007; *Keller et al.*, 2008; *Moser and Ekstrom*, 2010]. Traditional approaches in multivariate analysis literature do not fully address the sources of uncertainties. We argue that the proposed approach in this paper offers an avenue to account for the underlying uncertainties in multi-hazard assessment.

5 Conclusions

It is important to consider the compounding effects of multiple inter-dependent extremes or drivers to accurately characterize the underlying hazard. In this paper, we discuss multiple design scenarios and hazard assessment frameworks associated with compound events, and their uncertainties based on a multivariate framework. Here we summarize our conclusions:

- The choice of copula is crucially decisive for multivariate hazard assessment and design quantile estimation (Figure 3), which has not received the attention it deserves in the literature. In most hydrology and climate studies, only few models (typically a handful) are tested for fitting and multivariate analysis. In our coastal flooding example, the most likely compound extreme scenario varies in a range that equals to 56% of the mean annual maximum river discharge and 178% of the average water level, for different copula models. We recommend using a wide range of models with different characteristics to ensure the fitted multivariate model is representative.
- Translation of modeling uncertainties into multivariate design quantiles is a critical aspect of multivariate analyses. While some copula models show a relatively confined level of uncertainty (e.g., Joe copula with the most likely design quantiles' ranges equal to 6% and 3% of the mean annual maximum river discharge and the mean water level, respectively), others display a large range of uncertainty in their design quantiles (e.g., Gaussian copula with the most likely design quantiles' ranges equal to 337% and 97% of the mean annual maximum river discharge and the mean water level, respectively). For different case studies and data sets, the best choice of model with response to uncertainty bounds may change.
- We also note that the choice of marginal distribution plays an important role in determining the design quantiles. Figure S2 (SI) shows significant divergence between different marginal distributions representing river discharge (A) and water level (B). But this issue is not limited to multivariate analysis and the same applies to univariate applications.

Moreover, in this paper we introduce the concept of a multi-hazard expected event, with threshold quantiles derived based on the notion of weighted average of possible events. This multivariate event shows less sensitivity to the choice of copula. In our coastal flooding case study, for example, such threshold scenario ranges between [4490.77 m^3/s - 5042.82 m^3/s] for annual maximum river discharge and [1.58 m - 1.72 m] for water level, extents of which equals to 16% and 10% of the mean annual maximum river discharge and the mean water level, respectively.

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