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# Multivariate Standardized Drought Index: A parametric multi-index model

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ABSTRACT

Defining droughts based on a single variable/index (e.g., precipitation, soil moisture, or runoff) may not be sufficient for reliable risk assessment and decision-making. In this paper, a multivariate, multi-index drought-modeling approach is proposed using the concept of copulas. The proposed model, named Multivariate Standardized Drought Index (MSDI), probabilistically combines the Standardized Precipitation Index (SPI) and the Standardized Soil Moisture Index (SSI) for drought characterization. In other words, MSDI incorporates the meteorological and agricultural drought conditions for overall characterization of drought. In this study, the proposed MSDI is utilized to characterize the drought conditions over several Climate Divisions in California and North Carolina. The MSDI-based drought analyses are then compared with SPI and SSI. The results reveal that MSDI indicates the drought onset and termination based on the combination of SPI and SSI, with onset being dominated by SPI and drought persistence being more similar to SSI behavior. Overall, the proposed MSDI is shown to be a reasonable model for combining multiple indices probabilistically.



## 1. Introduction

Droughts are common climatic extremes that often spread across large spatial and time scales [27]. Historically, droughts affect more people across the globe than any other climate extremes [45]. The economic damage of droughts across the United States on average is estimated as $6–8 billion annually [12]; hence, monitoring and understanding the effects of droughts on water resource systems are essential to hazard preparedness and sustainable development.

The drought phenomenon is usually described using drought detection and monitoring indices. Typically, droughts are categorized into four major classes: meteorological, agricultural, hydrological, and socio-economical [18]. Meteorological drought is identified by lack of precipitation as the main indicator, while agricultural drought is related to the total soil moisture deficit. Hydrological drought, on the other hand, is characterized by a shortage of streamflow, as well as ground-water supplies. Several indices have been developed for drought monitoring based on different variables, such as precipitation, soil moisture, and runoff [24]. For example, the Palmer drought severity index (PDSI, [29]), derived from precipitation and temperature, has been widely used for drought characterization [8,7,35]. Mckee et al. [23] proposed the Standardized Precipitation Index (SPI) as a drought indicator for meteorological drought monitoring and analysis, which is recommended by the World Meteorological Organization (WMO) as a standard drought-monitoring index [17]. Given its simplicity and temporal flexibility, the SPI has been commonly used in numerous publications [26,38,41]. Other drought indices, such as the Crop Moisture Index, the Vegetation Drought Response Index (VegDRI), or the Standardized Precipitation Evapotranspiration Index (SPEI), have also been developed for drought monitoring [6,18,30,43].

A variety of studies have been conducted to evaluate the suitability of drought indices for different applications. Guttman [16] compared the PDSI and SPI and reported that the PDSI varied from site to site throughout the US with complex structure and long memory, while the SPI did not vary from site to site and was an easily interpreted, simple moving average process. Keyantash and Dracup [21] evaluated the most prominent indices for different forms of drought based on a weighted set of six evaluation criteria (e.g., robustness, tractability). Their results showed that rainfall deciles (followed by SPI with very close scores), total water deficit, and computed soil moisture were the overall superior drought indices for the meteorological, hydrological, and agricultural droughts, respectively. For monitoring meteorological drought, [31] suggested that the SPI and deciles (percentiles) were the most suitable indices.

Drought analyses based on a single variable (or indicator) may not be sufficient because drought phenomena are related to multiple variables (e.g., precipitation, runoff, and soil moisture). A meteorological drought (deficit in precipitation) may not lead to an

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agricultural drought (deficit in soil moisture), for example, in tropical regions where the average precipitation is relatively high. A complete analysis of drought events necessitates joint analyses of rainfall, runoff, and soil moisture conditions [9]. To characterize the overall drought condition, several joint drought indices have been proposed. Keyantash and Dracup [22] proposed an aggregate joint index that considers all physical forms of drought (meteorological, hydrological, and agricultural) through the selection of drought variables that are related to each drought type. Kao and Govindaraju [20] developed a copula-based joint index with Kendall distribution to characterize drought from precipitation and streamflow. Vicente-Serrano et al. [43] proposed the Standardized Precipitation Evapotranspiration Index based on precipitation and temperature data that combine multi-scalar characters with the capacity to include the effects of temperature variability on drought assessment.

In this paper, a multivariate, multi-index drought-modeling approach is proposed to combine the drought information from precipitation and soil moisture using the joint distribution function of the two variables. The proposed multi-index drought modeling framework is basically the extended version of the commonly used Standardized Precipitation Index (SPI), developed by [23], that incorporates soil moisture in addition to precipitation. Similar to the SPI, the suggested multi-index drought model is capable of characterizing drought conditions at different time scales. In this study, the proposed multi-index approach is utilized to characterize the drought conditions over several Climate Divisions in California and North Carolina. The results are then compared with SPI and SSI.

This paper is organized into five sections. The methodology is described in Section 2, while Section 3 provides a synthetic example and provides a discussion on how to interpret the suggested multivariate multi-index drought model. The application of the proposed framework is demonstrated in Section 4, followed by the summary and conclusion in Section 5.

## 2. Methodology

Motivated by the commonly used SPI developed by [23], a multi-index model can be developed through constructing the joint distribution function of two or more univariate drought variables (or indices). In this study, the Multivariate Standardized Drought Index (MSDI) is proposed by extending the univariate SPI through the joint distribution of precipitation and soil moisture for overall meteorological and agricultural drought characterization.

Copulas are functions that can be used to derive the joint distribution of two or more variables, regardless of their original marginal distributions. Assuming precipitation and soil moisture as random variables $X$ and $Y$, respectively, the joint distribution with the cumulative joint probability $p$ can be expressed with a copula $C$ as [28,40]:

$$P(X \leqslant x, Y \leqslant y) = C[F(X), G(Y)] = p \tag{1}$$

where $C$ is the copula, and $F(X)$ and $G(Y)$ are the marginal cumulative distribution functions of random variables $X$ and $Y$, respectively. The copula $C$ offers the flexibility to construct the joint distribution of random variables in terms of their marginal distributions. The application of copulas in modeling (nonlinear) dependence structures of multivariate data has become popular in hydrological and climatological studies, such as multivariate frequency analysis, risk assessment, drought modeling, and geostatistical interpolation [1,2,5,11,14,32,34,36].

There are a wide variety of copula families that have been developed/used to model different dependence structures of random variables [3,4,33]. For example, the Frank copula offers a symmetric dependence structure, while Gumbel and Clayton copulas exhibit asymmetric dependence structures [33,42]. The Frank copula, for example, can be expressed as [28,33]:

$$C(u, v) = -\frac{1}{\theta} \ln\left[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}\right] \tag{2}$$

where $\theta$ is the parameter, and $u$ and $v$ are the marginal cumulative probabilities of precipitation and soil moisture, respectively. The parameter $\theta$ can be estimated from Kendall's rank correlation $\tau$ [13]:

$$\tau = 1 + 4[D(\theta) - 1]/\theta \tag{3}$$

where $D(\theta)$ is expressed as:

$$D(\theta) = \frac{1}{\theta}\int_0^{\theta} \frac{t}{\exp(t) - 1} dt \tag{4}$$

where $t$ is the integration variable. The choice of copula family is discussed in Section 4. For detailed descriptions of different copula functions the interested reader is referred to [19,28].

From the cumulative joint probability $p$ shown in Equation (1), the Multivariate Standardized Drought Index (MSDI) can then be defined as:

$$\mathrm{MSDI} = \varphi^{-1}(p) \tag{5}$$

where $\varphi$ is the standard normal distribution function. Equation (5) transforms the joint probability to the MSDI that is located in the same space as the original SPI and allows cross-comparison of different drought indices. The procedure to develop the SPI can also be applied to other variables such as soil moisture and runoff [23,25,37]. In this study, we use the Standardized Soil Moisture Index (SSI) for cross-comparison. The proposed MSDI incorporates the overall drought conditions reflected from precipitation and soil moisture. Similar to the original SPI, a sequence of negative MSDIs indicates that the climate condition is dry (drought), while a sequence of positive MSDIs represents a wet climate condition. MSDI near zero refers to normal climate conditions.

Kao and Govindaraju [20] first described the concept of using the joint cumulative probability as the overall drought indicator and proposed the joint deficit index based on the Kendall distribution. In this study, we use the joint cumulative probability to construct the MSDI as an extension to the original SPI developed by McKee et al. [23]. Therefore, the proposed MSDI bears a close resemblance to the SPI based on the fact that it can be used to monitor droughts at different time scales (e.g., 1-, 3-, 6-month).

## 3. Interpretation of MSDI

In this section, we demonstrate the properties of MSDI through a numerical example. In Section 4, the process of selecting a copula family for deriving MSDI is discussed in detail. For now, assume that Frank copula is selected to construct the joint distribution of precipitation and soil moisture and to derive the MSDI.

The joint cumulative probability of precipitation and soil moisture given in Eq. (1) based on the Frank copula is displayed in Fig. 1. In this graph, 0.2 (or 20th percentile) of precipitation and soil moisture, for example, corresponds to SPI and SSI equal to −0.8. The MSDI contours for different combinations of precipitation and soil moisture percentiles are also shown in Fig. 1. Assume a certain drought threshold of 20th percentile precipitation and soil moisture (see lines L1 and L2 in Fig. 1). Consequently, four areas, A1–A4, are defined in the probability space by lines L1, L2, and the axes. The four areas (A1–A4) define different combinations of drought conditions indicated by precipitation and soil moisture. For a better illustration, the four points representing the four combinations of precipitation and soil moisture with different proba-

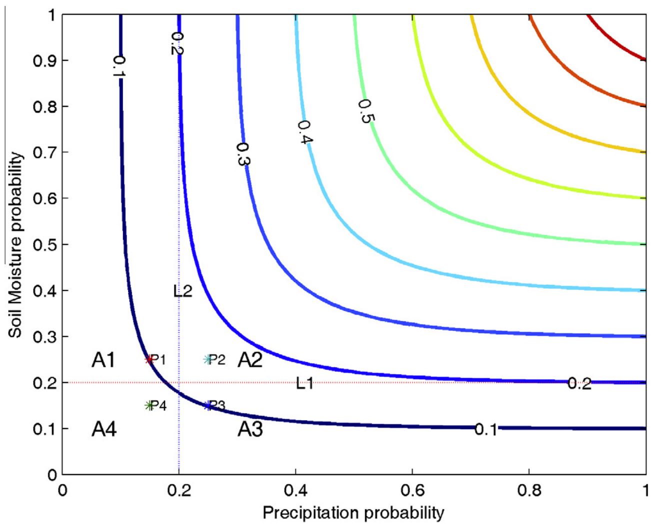


**Fig. 1.** A numerical example to describe the properties of MSDI.

bilities (or percentiles) are plotted in Fig. 1: P1 (0.15, 0.25), P2 (0.25, 0.25), P3 (0.25, 0.15), and P4 (0.15, 0.15), which fall into areas A1–A4, respectively. Considering only 20th percentile of precipitation, points that fall within A1 and A4 are in drought condition. Similarly, based on 20th percentile of soil moisture only, points that fall within areas A3 and A4 are in drought. A close look at Fig. 1 reveals that A1 corresponds to the condition in which precipitation shows drought, while soil moisture does not (e.g., P1 (0.15, 0.25)). On the contrary, A3 refers to the case in which there is a deficit in soil moisture, but not precipitation (e.g., P3 (0.25, 0.15)).

The area between 0.2 MSDI, L1 and L2, is shown as A2 in Fig. 1. Based on the 0.2-contour line of MSDI, all points that fall in A1 + A2 + A3 + A4 indicate drought, meaning that, for the same percentile (e.g., 20th), MSDI constitutes a larger probability space below the same percentile. For this reason, MSDI has a higher chance of detecting a drought based on the states of two variables (here, precipitation and soil moisture). Assuming a certain alarm threshold of 0.2, one can see that, for P2 (0.25, 0.25) MSDI indicates drought because both variables are low, although individual variables are still 0.25 (25th percentile).

Area A4 shown in Fig. 1 corresponds to the condition that both the soil moisture and precipitation fall below an alarm threshold (e.g., P4 (0.15, 0.15)). Both SPI and SSI will identify such points as a drought event, assuming an alarm threshold of 0.2 (20th percentile). As shown, P4 not only falls below the 0.20 contour of MSDI, it might even fall below a smaller value (here, 0.1). This indicates that, if both variables fall below an alarm threshold, MSDI will lead to a more severe drought condition than either SPI or SSI.

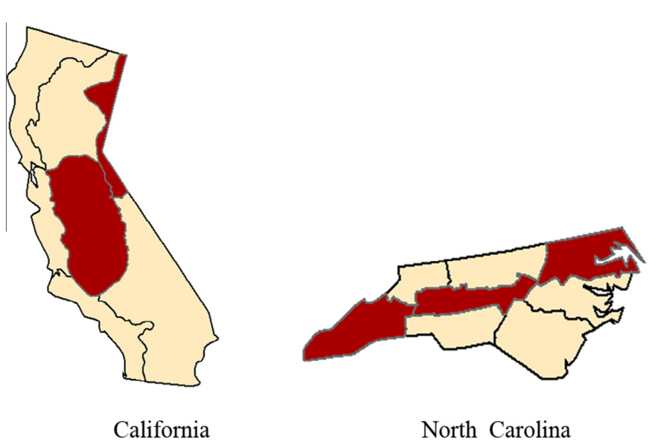


**Fig. 2.** Locations of the selected Climate Divisions in California and North Carolina.

## 4. Results

### 4.1. Data description

Monthly precipitation and soil moisture data are processed to illustrate the application of the proposed MSDI. Monthly precipitation and soil moisture data for the same period 1932–2009 were obtained from the Climate Prediction Center (CPC). The soil moisture data set is based on a one-layer water-budget soil moisture model available for the entire US [10]. As shown in Fig. 2, two Climate Divisions (3, 5) in California and three Climate Divisions (1, 4, and 8) in North Carolina are used as case study sites. The Climate Divisions represent different climate, different land-use and topographical conditions in the western and eastern United States. In California, Climate Division 3 is primarily an agricultural area, whereas Climate Division 5 is an inland mountainous are. Climate Divisions 1, 4, and 8 in North Carolina are mountainous, semi-urban, and coastal, respectively.

### 4.2. Copula-based joint distribution

Following the procedure presented by [23], the standard precipitation index (SPI) and standard soil moisture index (SSI) are derived using the precipitation and soil moisture data sets discussed above. In order to investigate the MSDI at different time scales (i.e., 3-, 6- and 12-month), SPI and SSI are computed for the same durations and used for cross-comparison.

Three copulas, namely, Clayton, Frank, and Gumbel, are used to derive the joint probability distribution of precipitation and soil moisture. The Cramér–von Mises statistic ($S_n$) and Kolmogorov–Smirnov statistic ($T_n$) are used for goodness-of-fit tests to assess the performance of different copulas in modeling the dependence structure between precipitation and soil moisture [13,15]. The *p*-values of statistics $S_n$ and $T_n$, based on a run of 5000 samples of each month for a 3-month duration for Climate Division 3, are shown in Table 1. For constructing the joint distribution, a copula cannot be rejected if the corresponding *p*-value is equal to or higher than 0.05 (5% significance level). The goodness-of-fit test of the copula is performed at a monthly scale in order to be consistent with SPI and SSI analyses. The tables indicate that most months can be modeled using the Frank and Gumbel copulas, while Clayton occasionally fits as well (see underlined *p*-values in Table 1). When several copula families fit the data, the one with the highest *p*-value is selected for deriving the joint distribution of precipitation and soil moisture.

### 4.3. Standardized precipitation and soil moisture indices

The original SPI and SSI at different time durations (i.e., 3-, 6- and 12-month) are compared for Climate Divisions 3 and 5 displayed in Figs. 3 and 4, respectively. One can see that, while the two indices are generally consistent, there are discrepancies at several time steps between the two indices. In a few time steps, even the wet (positive) and dry (negative) signals are different. For example, in 1977, the 3-month SPI for Climate Division 3 shows a recovery from drought, while the SSI indicates that the drought continues for a few more months (see the top panel in Fig. 3). Such discrepancies could be due to abnormally high rainfall over a very short period of time, while most of the month remains dry (SPI > 0 and SSI < 0). Alternatively, a below-average rainfall distribution

**Table 1**
$p$-values for the goodness-of-fit tests $S_n$ and $T_n$ for deriving 3-month MSDI based on precipitation and soil moisture over Climate Division 3 in California (the underlined values for each month correspond to the suitable copula family).

| Copula | $p$-value | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Clayton | $S_n$ | 0.00 | 0.02 | 0.06 | 0.07 | 0.16 | 0.01 | 0.02 | 0.01 | 0.26 | 0.46 | 0.07 | 0.04 |
| Clayton | $T_n$ | 0.00 | 0.04 | 0.05 | 0.02 | 0.46 | 0.03 | 0.01 | 0.02 | 0.28 | 0.34 | 0.07 | 0.17 |
| Frank | $S_n$ | 0.46 | 0.33 | 0.40 | 0.33 | 0.57 | 0.42 | 0.13 | 0.61 | 0.80 | 0.01 | 0.32 | 0.84 |
| Frank | $T_n$ | 0.64 | 0.18 | 0.42 | 0.62 | 0.80 | 0.20 | 0.32 | 0.48 | 0.90 | 0.02 | 0.73 | 0.38 |
| Gumbel | $S_n$ | 0.27 | 0.39 | 0.23 | 0.85 | 0.82 | 0.35 | 0.35 | 0.28 | 0.95 | 0.01 | 0.84 | 0.41 |
| Gumbel | $T_n$ | 0.11 | 0.26 | 0.22 | 0.77 | 0.82 | 0.39 | 0.39 | 0.25 | 0.87 | 0.01 | 0.89 | 0.34 |

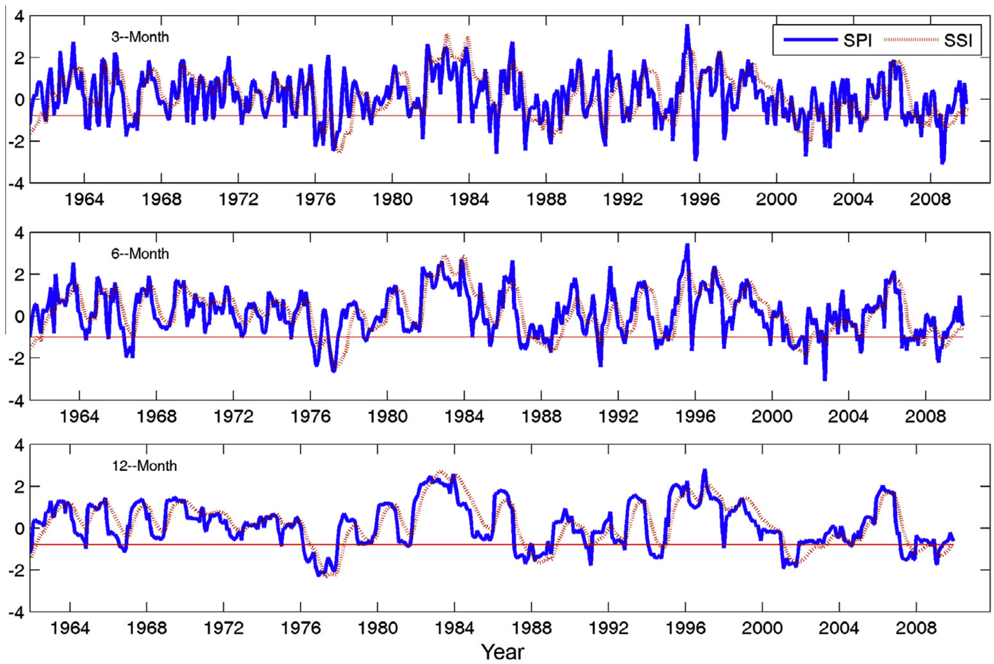


**Fig. 3.** Comparison of 3-, 6-, and 12-month SPI and SSI for Climate Division 3 in California ($y$-axes show the dimensionless values of SPI and SSI).

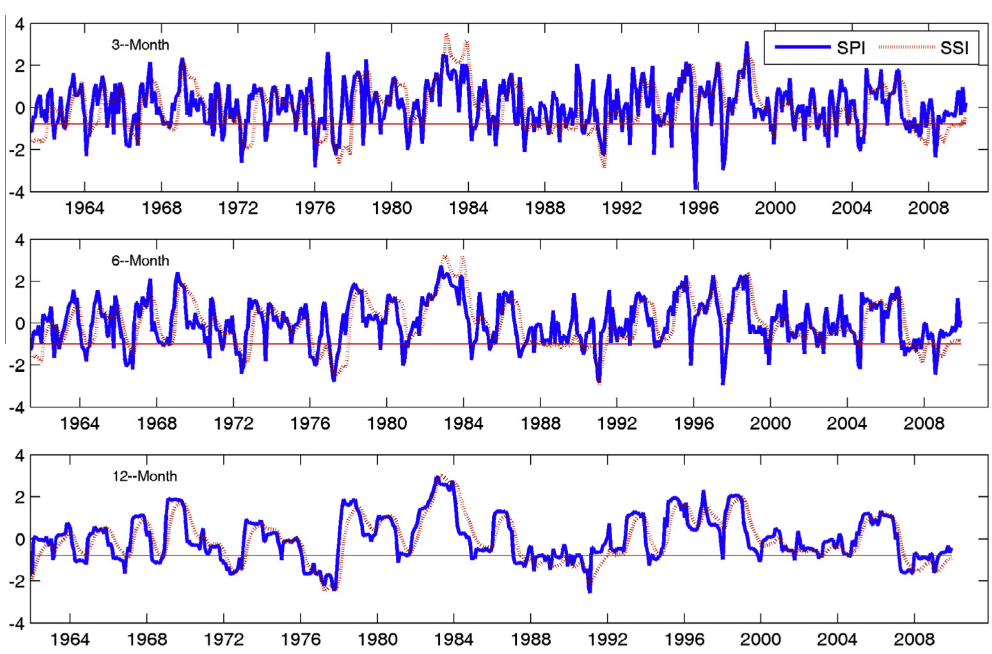


**Fig. 4.** Comparison of 3-, 6- and 12-month SPI and SSI for Climate Division 5 in California ($y$-axes show the dimensionless values of SPI and SSI).

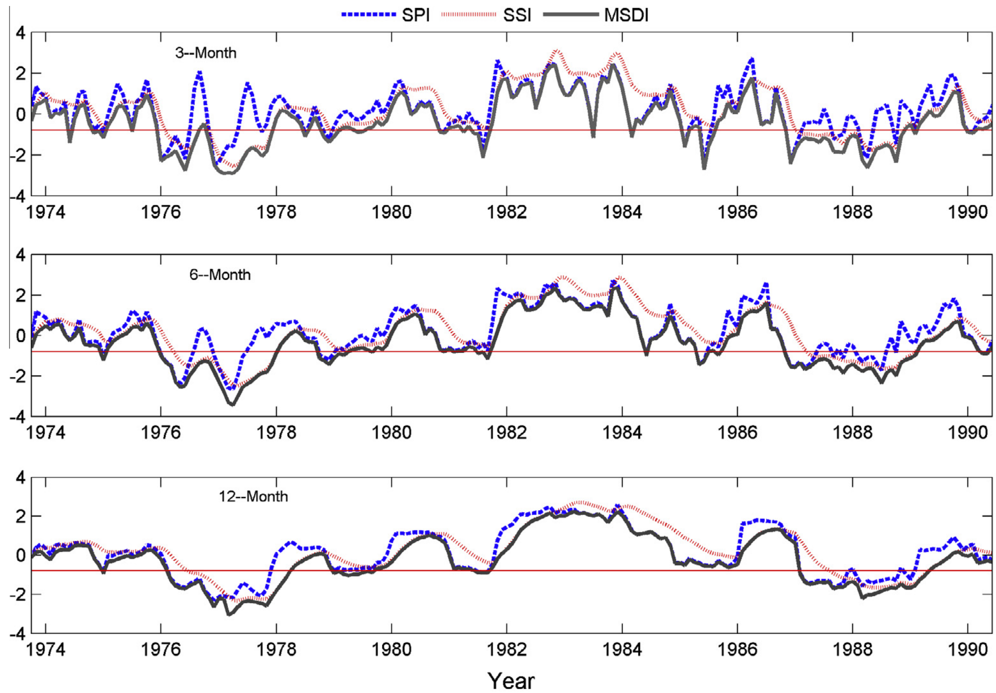


**Fig. 5.** Comparison of 3-, 6- and 12-month MSDI, SPI, and SSI for Climate Division 3 in California ($y$-axes show the dimensionless values of SPI, SSI and MSDI).

throughout a month, such that the soil stays wet (SPI < 0 and SSI > 0), could lead to opposite signs of SPI and SSI.

It is emphasized that as the drought duration increases (e.g., from 3- to 12-month), the differences between the SPI and SSI tend to decrease (see Figs. 3 and 4). For example, the 12-month SPI and SSI are more consistent compared to those of the 3-month or 6-month drought durations. It is worth mentioning that both SPI and SSI capture historical drought conditions, such as the California drought of 1976–1977. However, the SPI and SSI show different levels of severity (the horizontal lines in Figs. 3 and 4 represent the moderate drought threshold (severity of −0.8) for better comparison). Having different severities indicates that the risk assessment and return-period estimation using SPI and SSI will lead to different results. Furthermore, the results demonstrate that the estimated drought duration from SPI and SSI often varies considerably (e.g., 2001–2003 in Fig. 3(bottom) and 1991–1993 in Fig. 4(bottom)). These differences may lead to different definitions for drought onset and termination.

### 4.4. Multivariate Standardized Drought Index (MSDI)

We hypothesize that MSDI can provide a new perspective based on the joint probability distribution of precipitation and soil moisture. The 3, 6 and 12-month MSDI (solid line), SPI (dashed line), and SSI (dotted line) for the Climate Division 3 are plotted in Fig. 5 (for better visualization, only the results for 1974–1990 are shown). As an example, during 1976–1978, the 3-month SPI and SSI both show deficits in precipitation and soil moisture with different durations (Fig. 5(top)). The SPI captures the drought earlier than the SSI, and shows more variability compared to SSI. On the other hand, SSI indicates a longer drought compared to SPI, meaning it shows the drought persistence more reliably. The MSDI exhibits the drought onset similar to the SPI and drought persistence similar to SSI. The drought duration based on MSDI is similar to that of SSI and longer than the duration of the same event based on SPI. During this 2-year drought period, precipitation shows signals of drought recovery in mid-1976 and mid-1977 (see high values of SPI). However, the drought termination signals based on precipitation are temporary and primarily because of high variability of precipitation (see SSI which does not show much variability and shows the persistence of drought for the entire 2-year event). Therefore, describing droughts based on solely the state of the precipitation may be misleading at certain time-steps. Here, MSDI captures the drought as early as SPI (∼2 month before SSI shows the drought onset) and describes the drought development and termination based on the state of both precipitation and soil moisture. Therefore, a temporary change in one variable (e.g., precipitation) does not affect the MSDI.

The above example highlights an attractive property of MSDI which is describing the drought onset and persistence based on the states of multiple variables. When either precipitation or soil moisture indicates a drought event, MSDI will also show a drought event. Regarding the drought severity, the MSDI generally resembles the severity of the SPI or SSI whichever is lower. The MSDI will lead to even a more severe drought than SPI and SSI when both indices show deficits in both precipitation and soil moisture (e.g., during 1976–1978 drought in Fig. 5).

As shown in the 6-month SPI, SSI and MSDI in Fig. 5 (middle), for the drought that occurred at the end of 1981, the drought onset is first captured by the MSDI while neither precipitation nor soil moisture indicate the drought onset. However, both indicators are low and show signs of deficit. This is the case when the combined precipitation and soil moisture fall within the A2 area (see Fig. 1). For this reason, MSDI can improve earlier detection of droughts than each individual index. The presented results show that MSDI can combine the information from two indices and provide one measure of drought based on the states of both precipitation and soil moisture.

In order to further investigate the proposed methodology, MSDI is applied to three other Climate Divisions (1, 4, and 8) in North Carolina, which represent different land-use and topographical features (Division 1 is mountainous, 4 is semi-urban and 8 is coastal region) [39]. The 3-month SPI, SSI, and MSDI for the three Climate Divisions are shown in Fig. 6. As shown in Fig. 6(top), the onset of the drought that occurred between 1984 and 1986 in Climate Division 1 is first recognized by the 3-month SPI and MSDI. The SSI

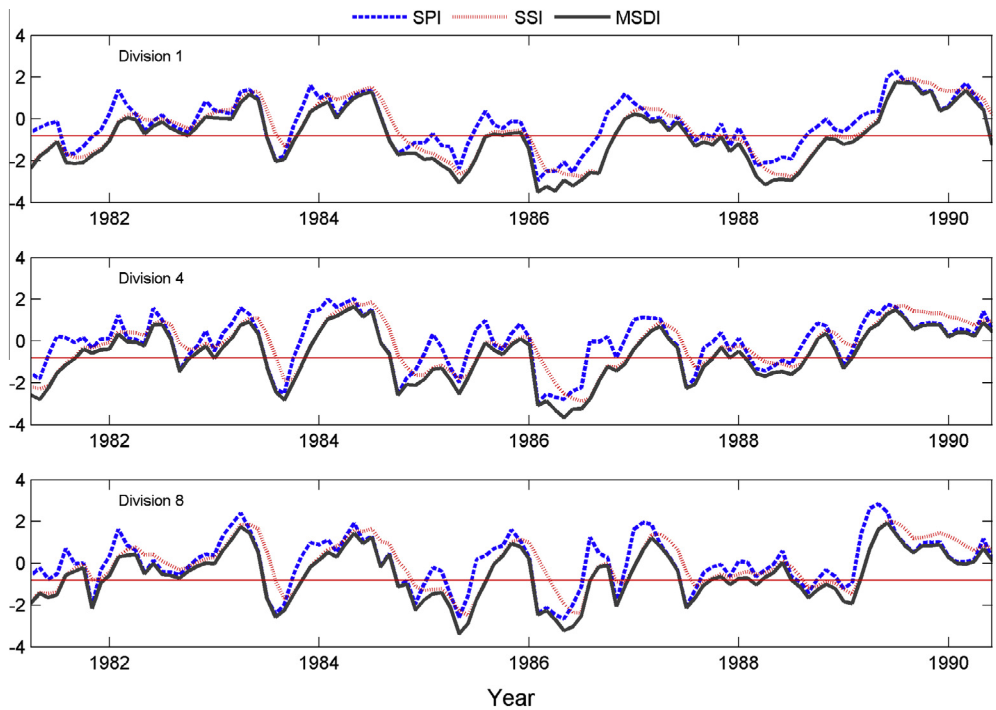


**Fig. 6.** Comparison of 3-month MSDI, SPI, and SSI for Climate Division 1, 4, and 8 in North Carolina ($y$-axes show the dimensionless values of SPI, SSI and MSDI).

does not capture this drought condition as early as SPI and MSDI. This is the case when the combination of precipitation and soil moisture falls in the A1 area displayed in Fig. 1. In this case, the MSDI, along with the SPI, identifies the drought, while SSI does not. As the drought develops, SSI also shows a deficit in soil moisture, which results in the MSDI showing an even higher severity level (or lower joint probability). This is the case where both precipitation and soil moisture highlight the drought (A4), and as such, MSDI exhibits an even more severe drought condition than SPI and SSI alone. As time evolves, precipitation returns to a near-normal level (above $-0.8$ thresholds) several months earlier than the SSI (and MSDI). This is the case where the combination of precipitation and soil moisture lies in the A3 area given in Fig. 1. As mentioned earlier, MSDI capture all conditions that fall in the four areas (A1–A4) highlighted in Fig. 1. Similar behavior can be observed in Climate 4 (the middle panel in Fig. 6) and Climate Division 8 (the bottom panel in Fig. 6) during this period. In summary, MSDI captures the drought onset as early as SPI (or earlier if both precipitation and soil moisture are low) and describes the drought persistence similar to SSI.

## 5. Summary and conclusions

The fundamental difference between droughts and other climate extremes such as floods and hurricanes lies in the fact that droughts occur over much longer time spans, and their onsets and terminations are difficult to identify [44]. While drought events are typically defined as periods with a sustained lack of water, depending on the region, indicator variable, and/or user requirement, they may be defined differently (e.g., lack of soil moisture, ground water, or precipitation). For this reason, providing reliable and relevant drought information based on multiple indicator or variables is important for overall characterization of drought.

In this paper, a multivariate, multi-index drought-modeling approach is proposed using the concept of copulas. The proposed model, named Multivariate Standardized Drought Index (MSDI), determines drought onset and termination based on the combination of SPI and SSI, with onset time being dominated by SPI and the persistence of droughts being more similar to SSI behavior. The properties of the MSDI that can be summarized as follows: (a) MSDI captures a drought condition indicated from either precipitation or soil moisture; (b) MSDI describes the drought onset as early as SPI, while it shows drought persistence similar to SSI; and (c) MSDI shows a more severe drought condition when both the precipitation and soil moisture exhibit a deficit. Notice that MSDI, similar to univariate SPI and SSI, provides probability of occurrence and thus can be used for risk analysis as well.

The proposed framework for creating multi-index drought models is rather general, and other indices can be integrated into MSDI. In the future, the authors will evaluate the integration of other indices, such as runoff or ground-water storage, to evaluate meteorological, agricultural, and hydrological droughts. The authors emphasize that drought information should be based on multiple sources of information and, for this reason, MSDI is not meant to replace SPI and SSI. Instead, we propose that MSDI be used as an additional source of information based on the joint probability of precipitation and soil moisture.

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